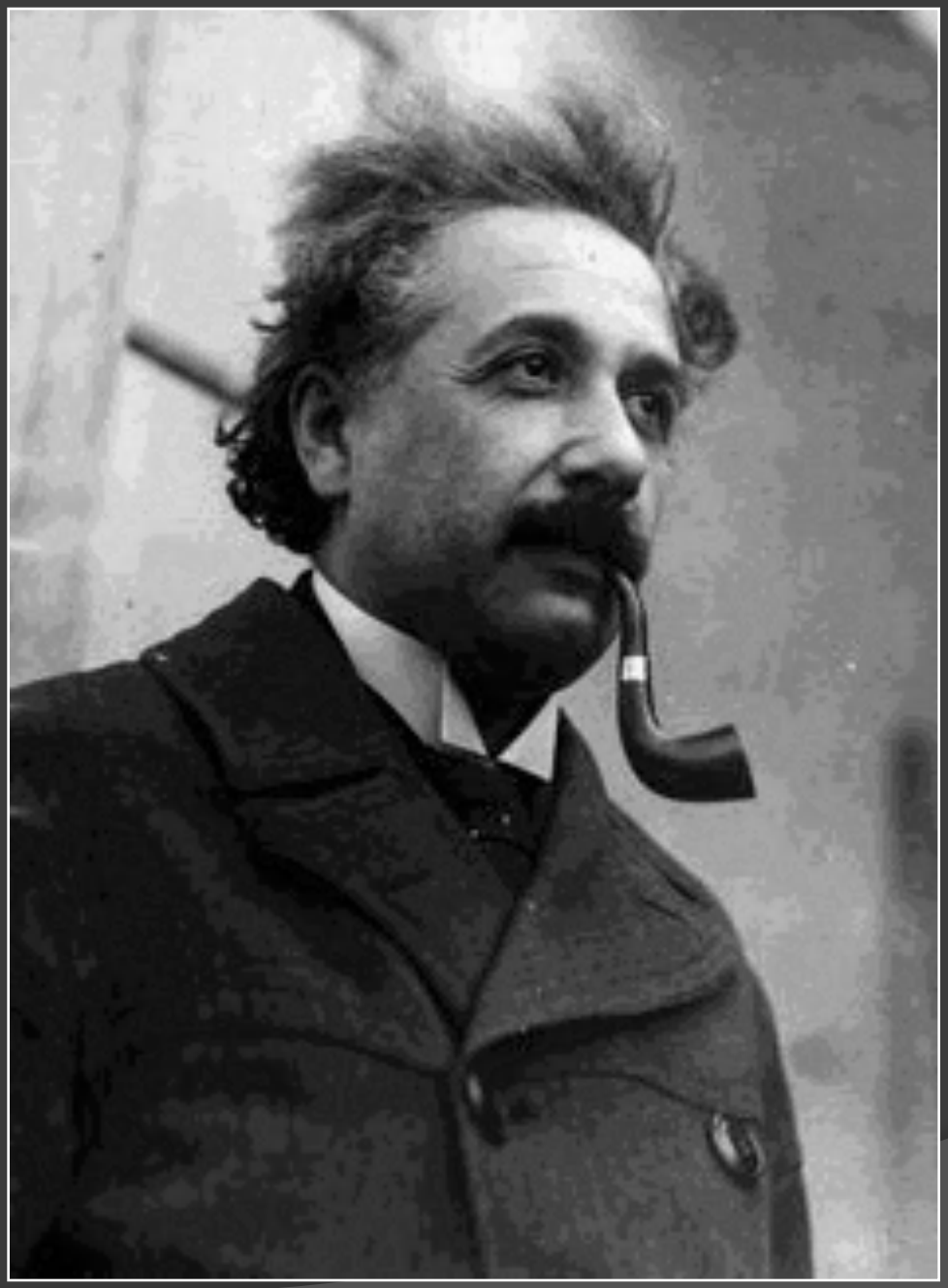
AN INTRODUCTION TO COSMOLOGY

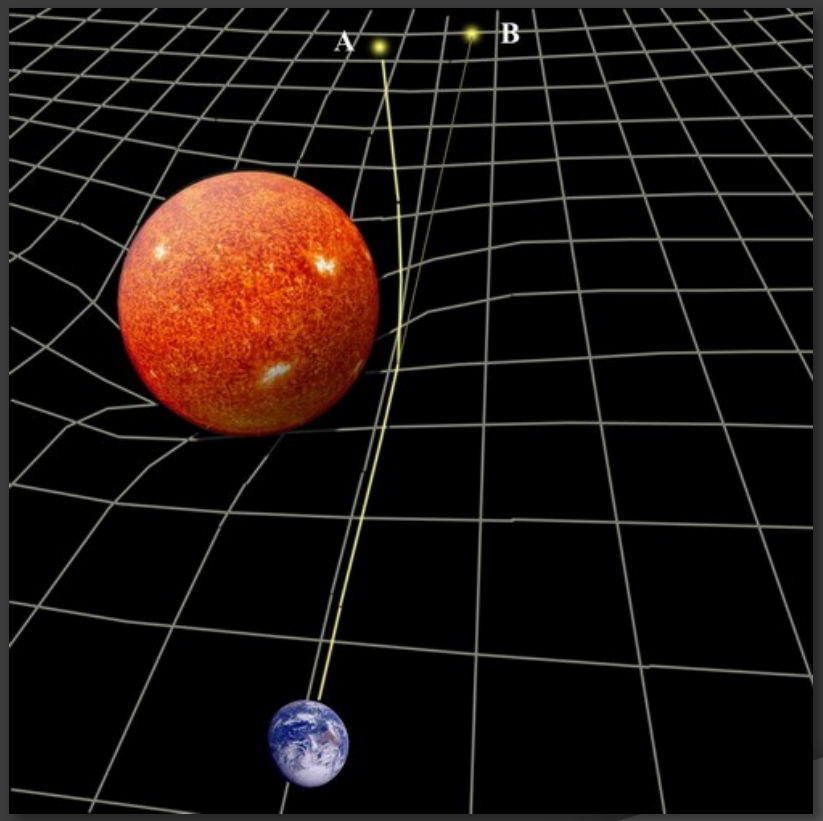
Dan Hooper – Fermilab

Undergraduate Lecture Series

June 2, 2020



$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi G}{c^4} T_{ij}$$



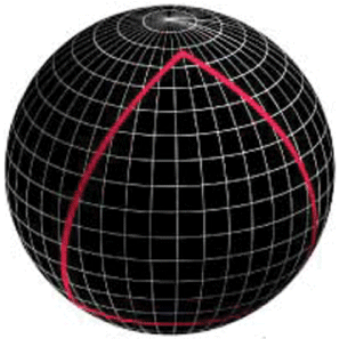
Cosmological Solution

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho$$

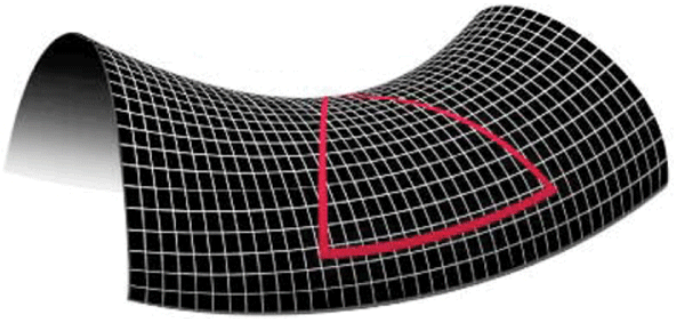
AND

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = -\frac{8\pi G}{c^2}p$$

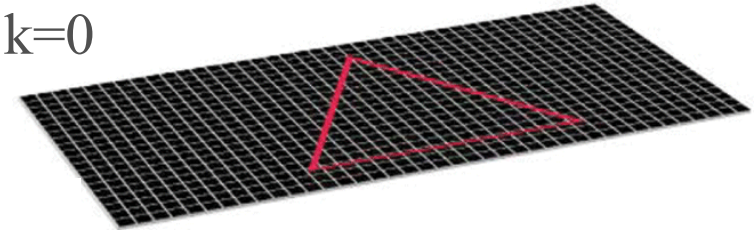
k=1



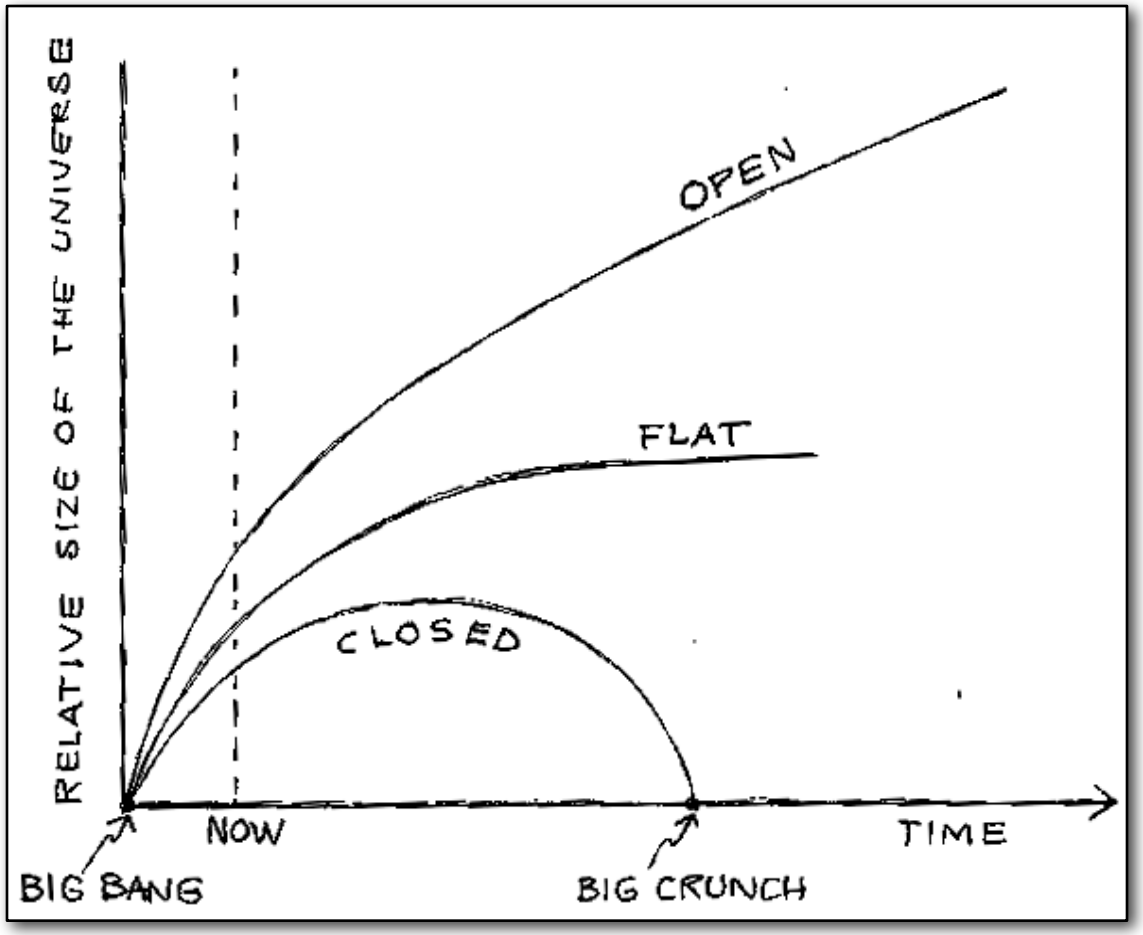
k=-1

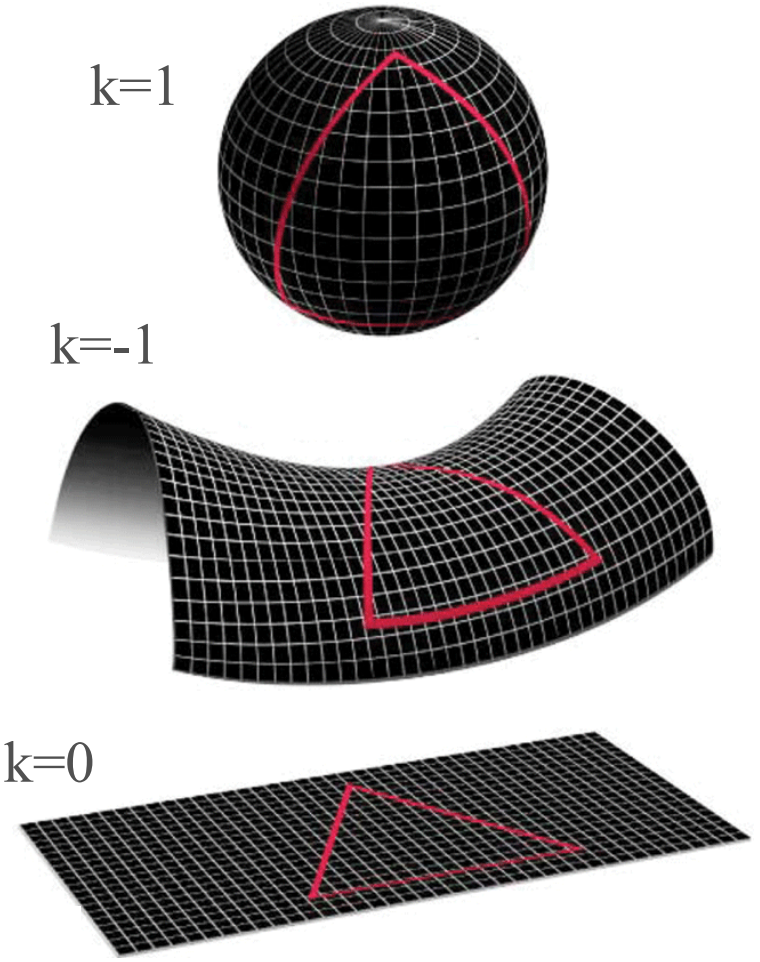


k=0



Cosmological Solution





$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

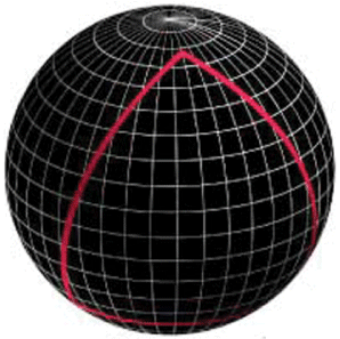
Solution with a "Cosmological Term"

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho$$

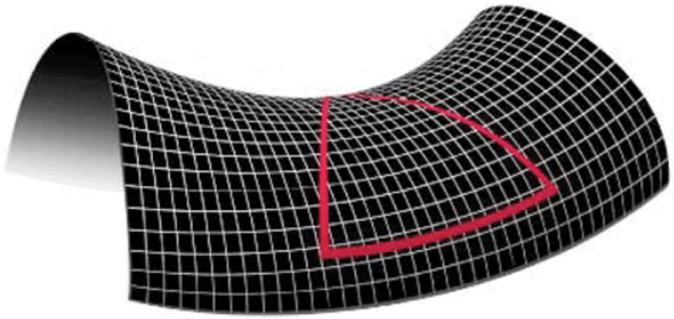
AND

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \Lambda c^2 = -\frac{8\pi G}{c^2}p$$

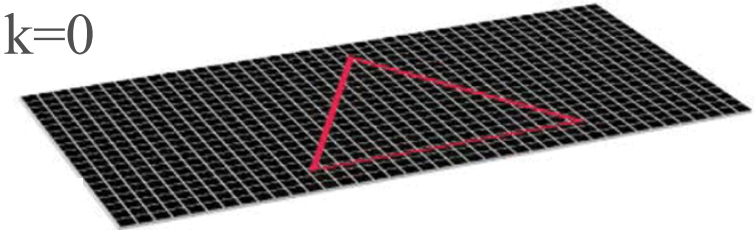
k=1



k=-1



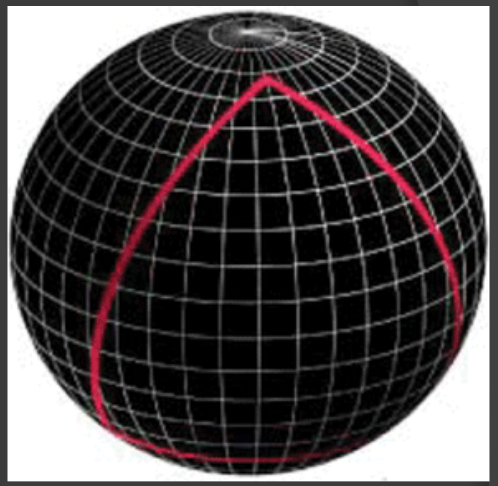
k=0



“Einstein’s World”

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3}$$

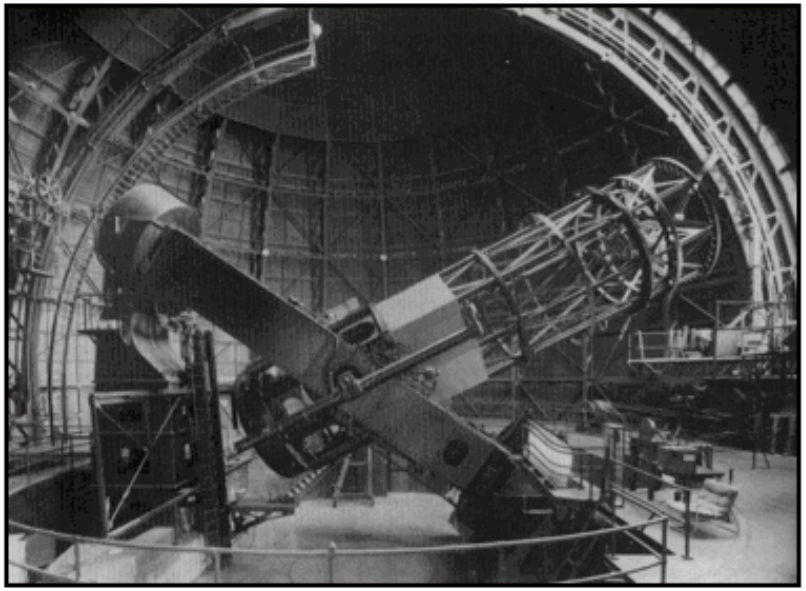
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$



Einstein found that for the choice of $\Lambda = 4\pi G\rho/c^2$, a static solution (with positive curvature, k=1) resulted

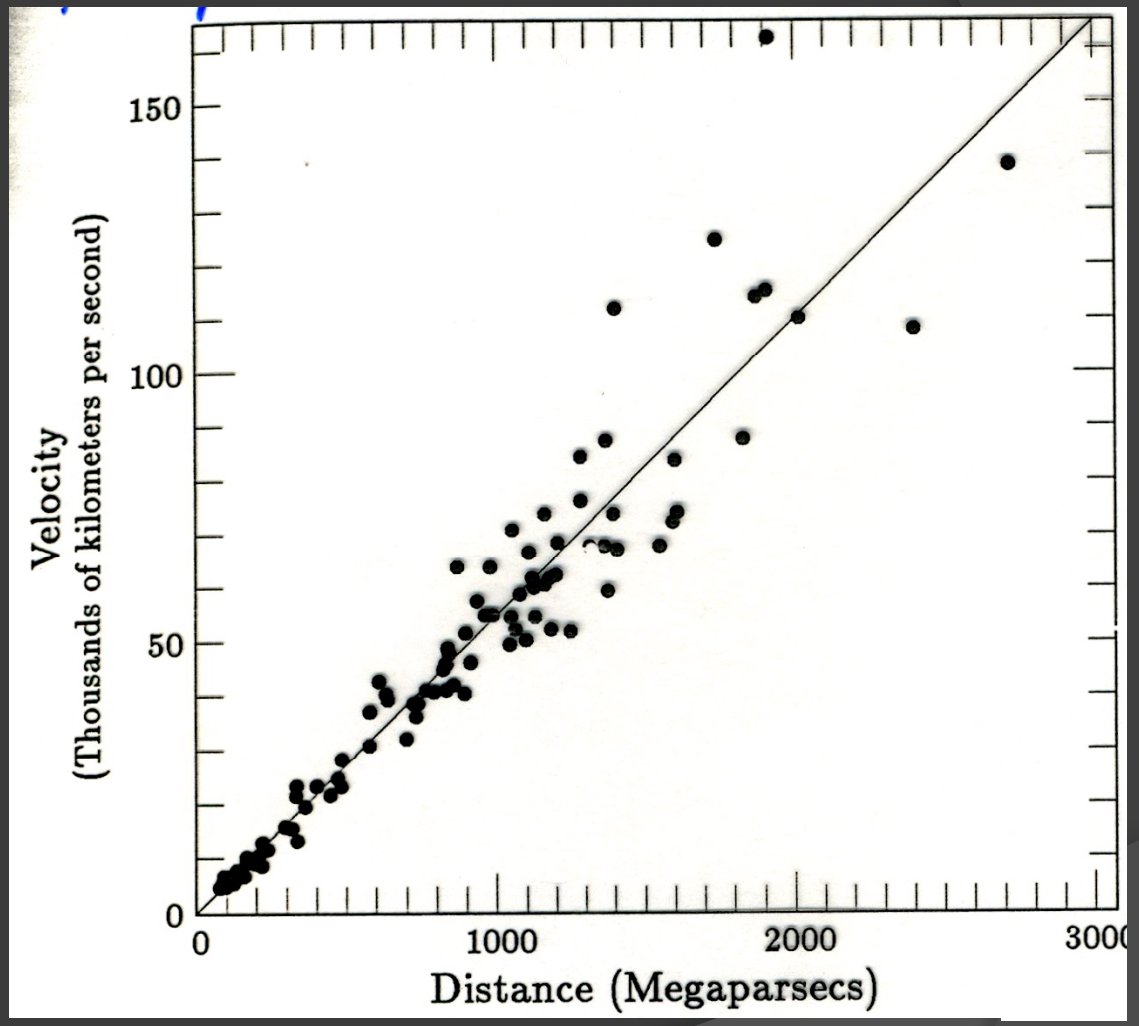
But this solution was unstable

Edwin Hubble
1889 – 1953



100 inch Mt Wilson Telescope

Milton Humason
1891 – 1972


Velocity
(Thousands of kilometers per second)
150
100
50
0
0
1000
2000
Distance (Megaparsecs)

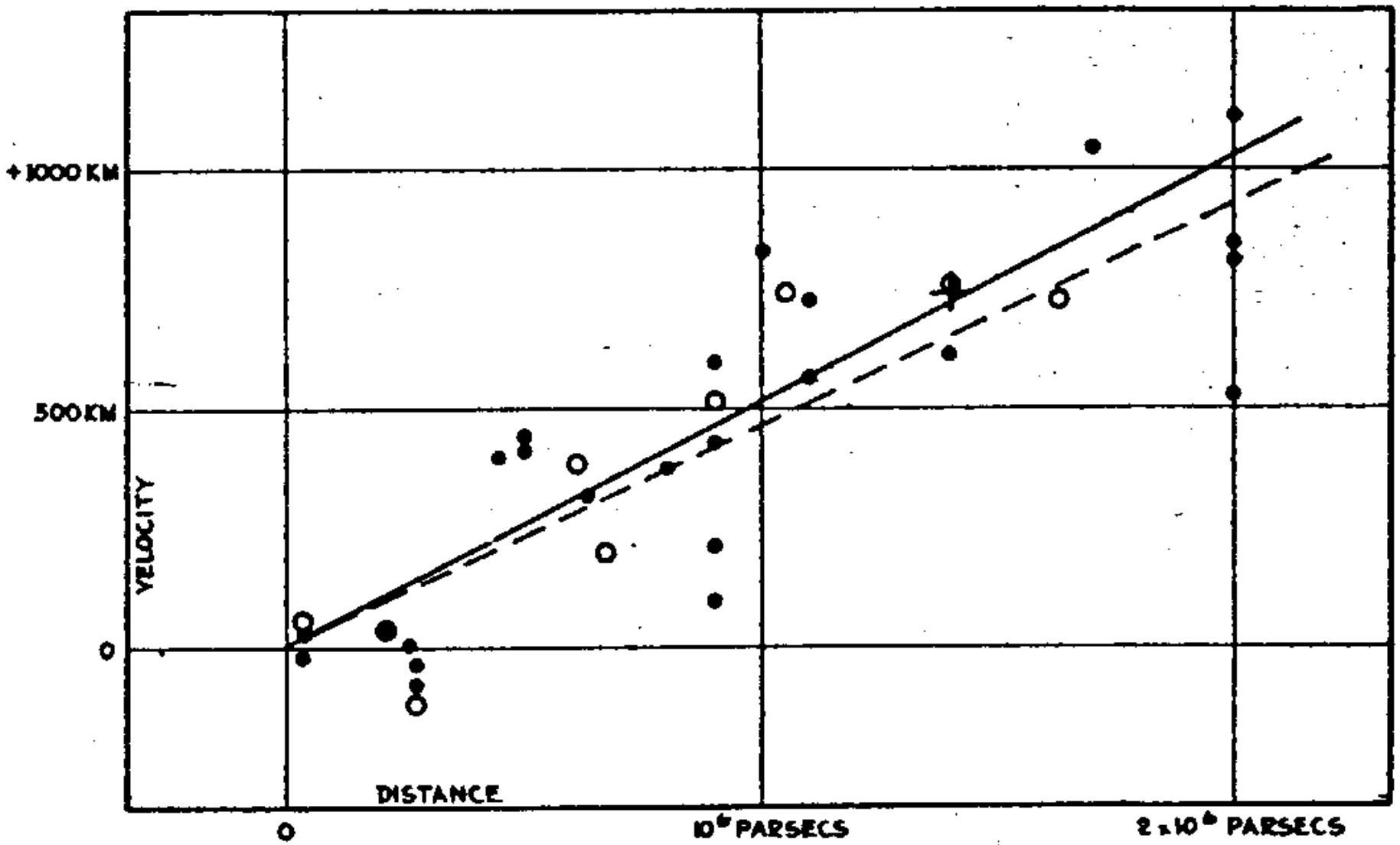
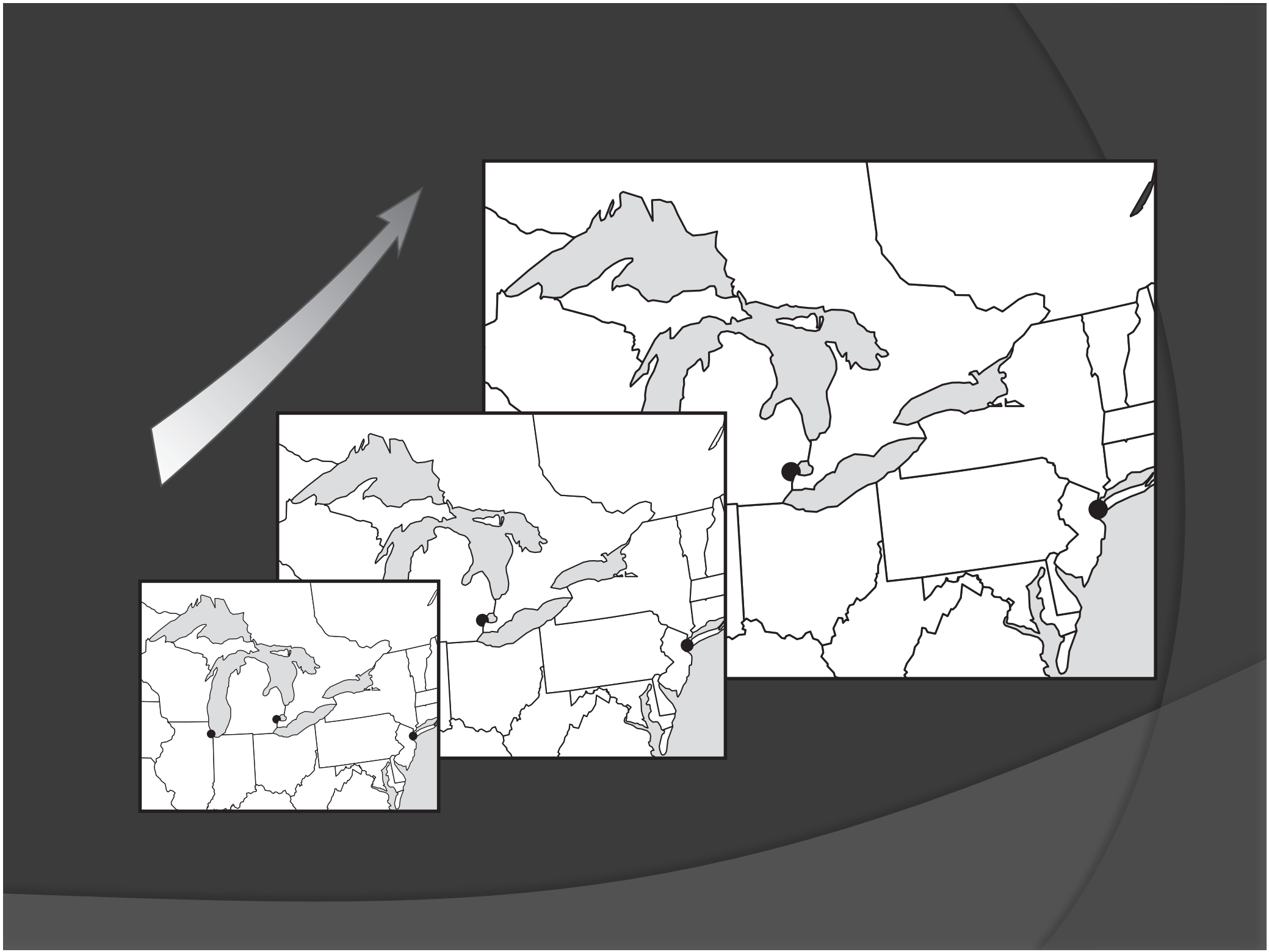
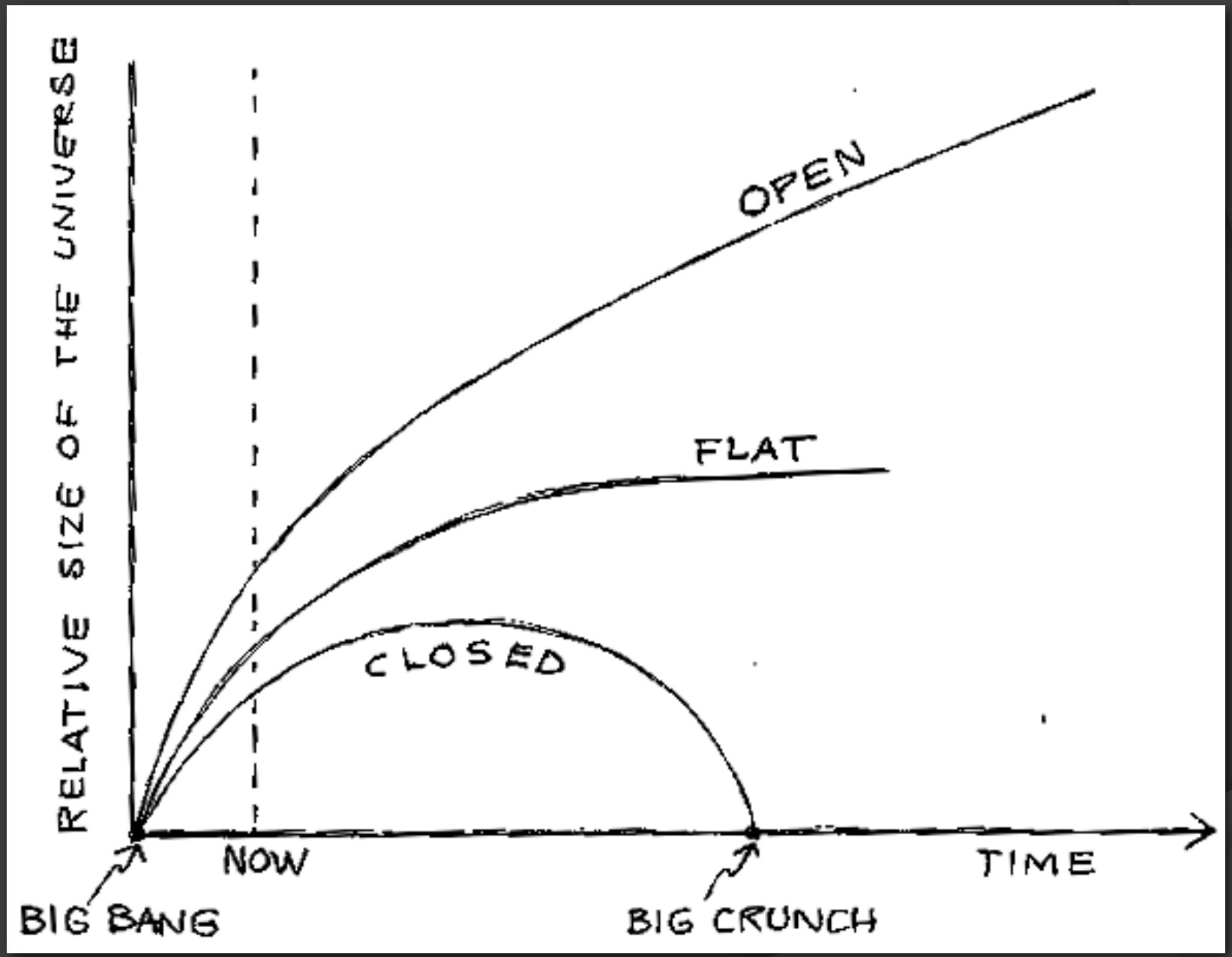
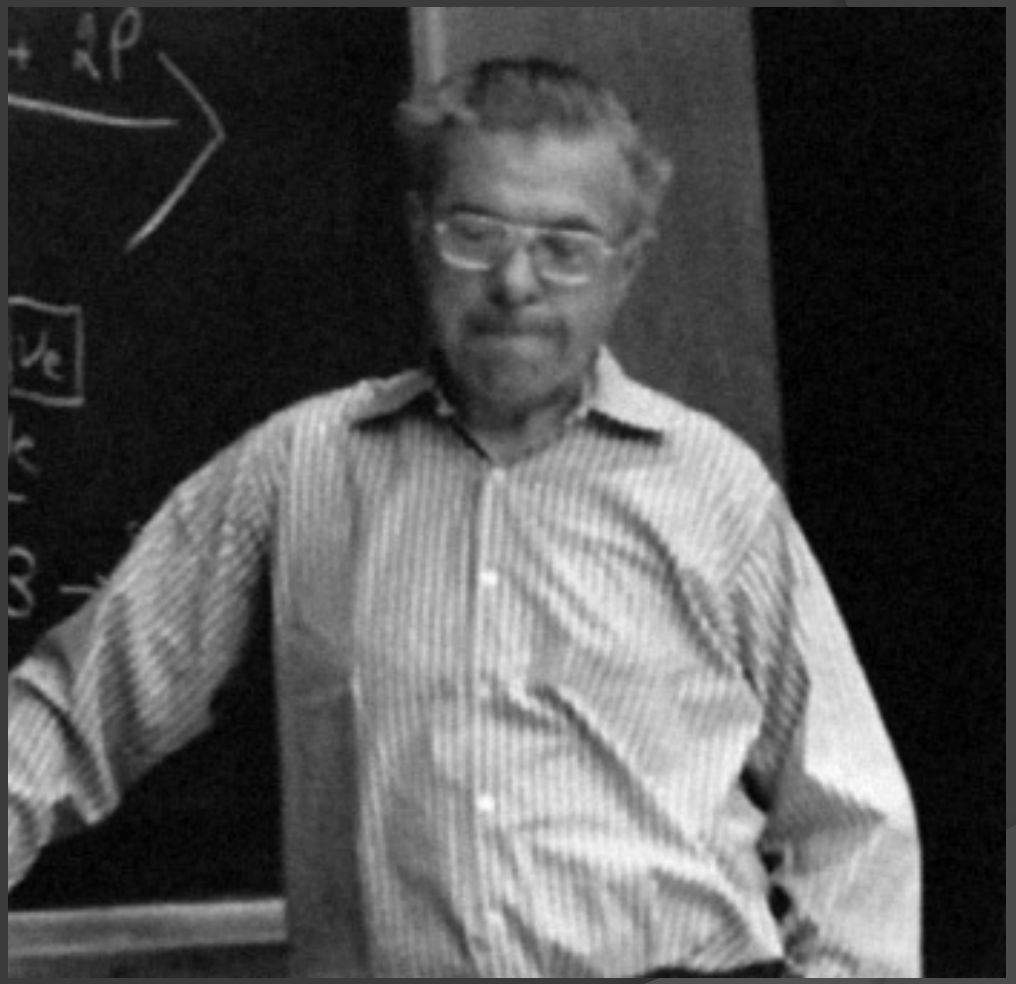

+1000 KM
500 KM
0
VELOCITY
DISTANCE
0
10⁶ PARSECS
2 x 10⁶ PARSECS

FIGURE 1




RELATIVE SIZE OF THE UNIVERSE
OPEN
FLAT
CLOSED
NOW
TIME
BIG BANG
BIG CRUNCH



Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length.

The Origin of Chemical Elements

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Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland
AND
H. BETHE
Cornell University, Ithaca, New York
AND
G. GAMOW
The George Washington University, Washington, D. C.
February 18, 1948

AS pointed out by one of us,[1] various nuclear species must have originated not as the result of an equilibrium corresponding to a certain temperature and density, but rather as a consequence of a continuous building-up process arrested by a rapid expansion and cooling of the primordial matter. According to this picture, we must imagine the early stage of matter as a highly compressed neutron gas (overheated neutral nuclear fluid) which started decaying into protons and electrons when the gas pressure fell down as the result of universal expansion. The radiative capture of the still remaining neutrons by the

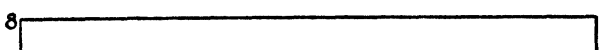
We may remark at first that the building-up process was apparently completed when the temperature of the neutron gas was still rather high, since otherwise the observed abundances would have been strongly affected by the resonances in the region of the slow neutrons. According to Hughes,[2] the neutron capture cross sections of various elements (for neutron energies of about 1 Mev) increase exponentially with atomic number halfway up the periodic system, remaining approximately constant for heavier elements.

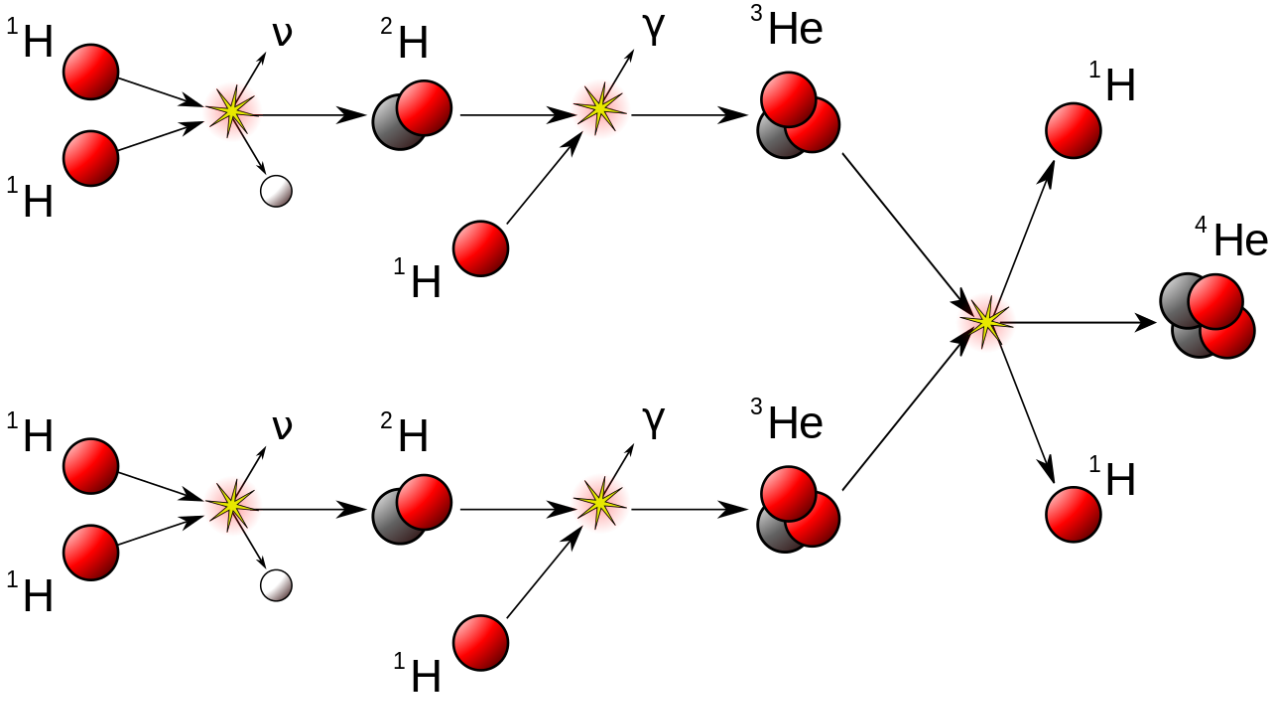
Using these cross sections, one finds by integrating Eqs. (1) as shown in Fig. 1 that the relative abundances of various nuclear species decrease rapidly for the lighter elements and remain approximately constant for the elements heavier than silver. In order to fit the calculated curve with the observed abundances[3] it is necessary to assume the integral of $\rho_n dt$ during the building-up period is equal to 5×10^4 g sec./cm^3.

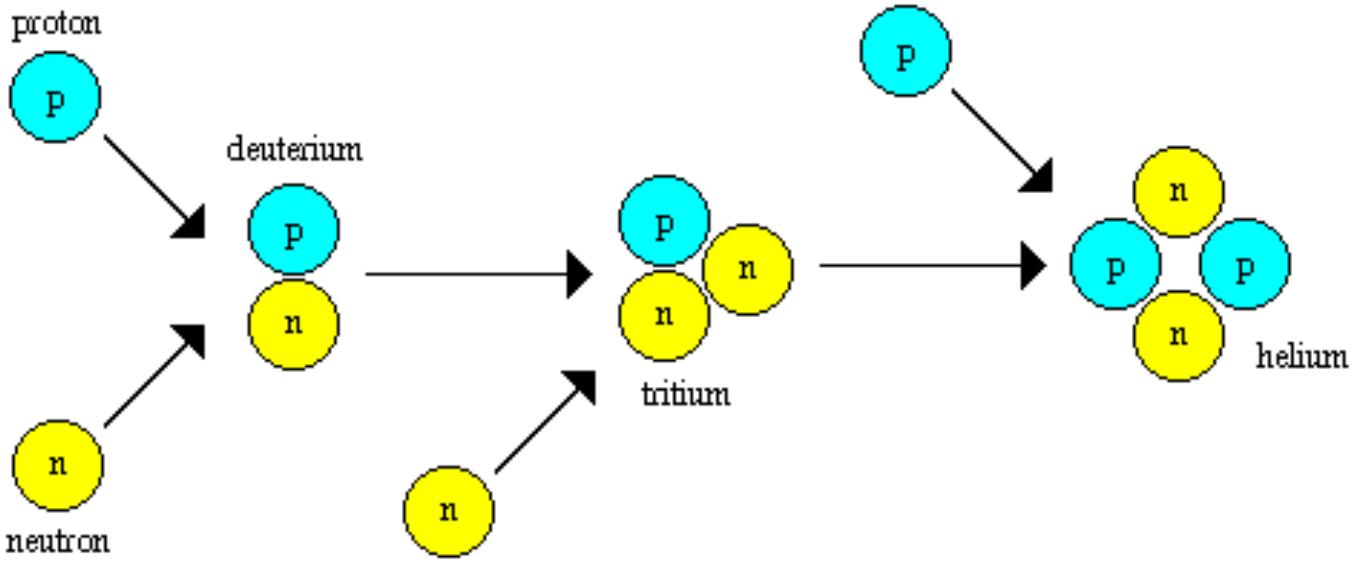
On the other hand, according to the relativistic theory of the expanding universe[4] the density dependence on time is given by $\rho \leqq 10^6/t^2$. Since the integral of this expression diverges at $t=0$, it is necessary to assume that the building-up process began at a certain time t_0, satisfying the relation:

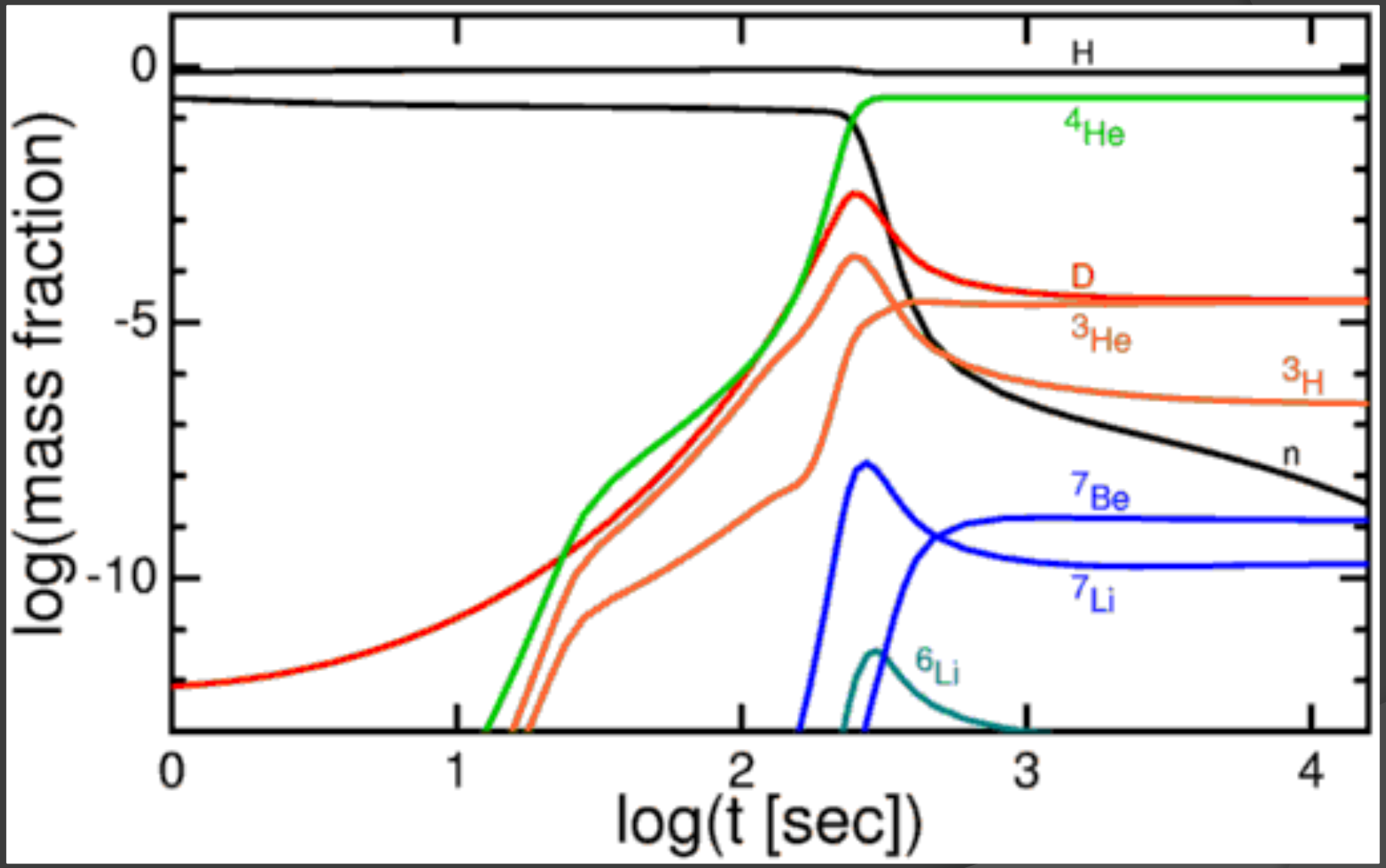
$$\int_{t_0}^{\infty} (10^6/t^2)dt \leqq 5\times10^4, \qquad (2)$$

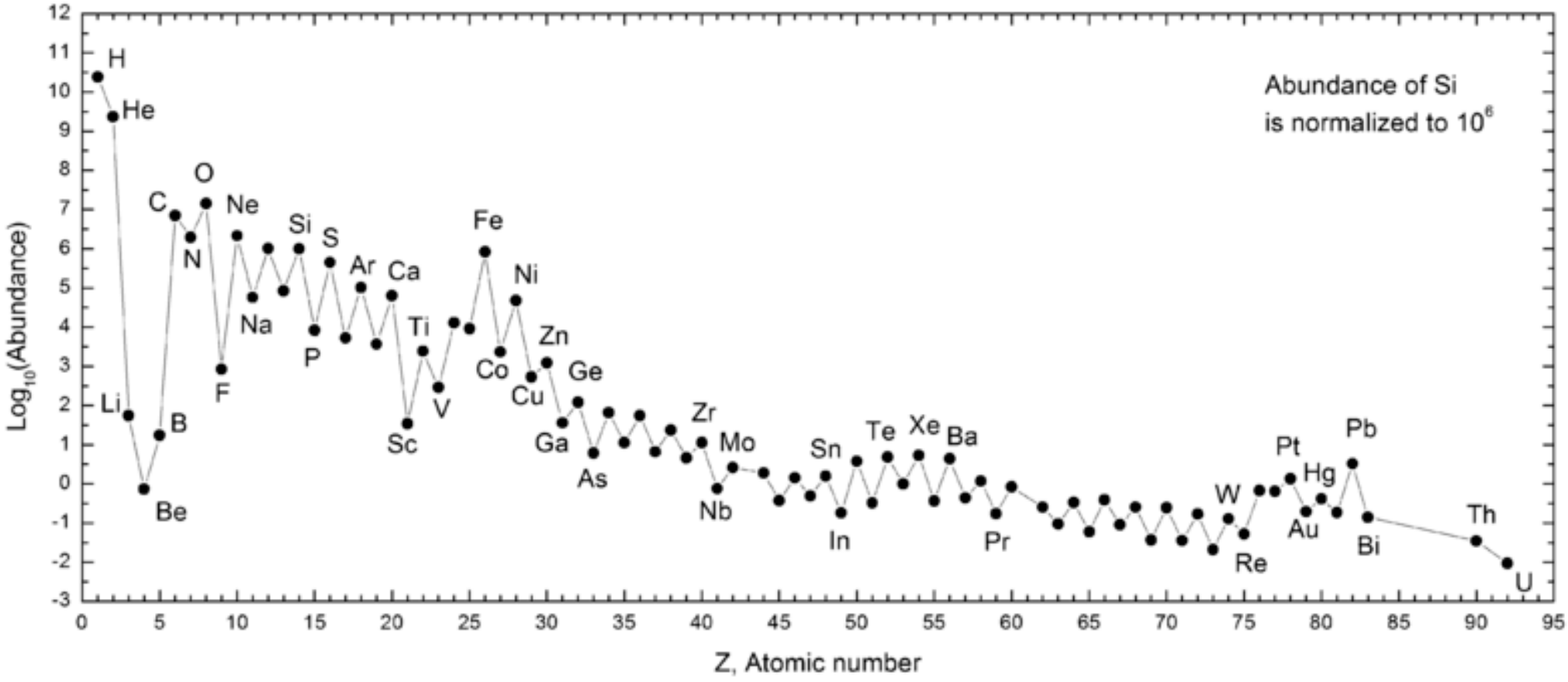
which gives us $t_0 \leqq 20$ sec. and $\rho_0 \leqq 2.5\times10^5$ g sec./cm^3. This result may have two meanings: (a) for the higher densities existing prior to that time the temperature of the neutron gas was so high that no aggregation was taking place, (b) the density of the universe never exceeded the value 2.5×10^3 g sec./cm^3 which can possibly be understood if we

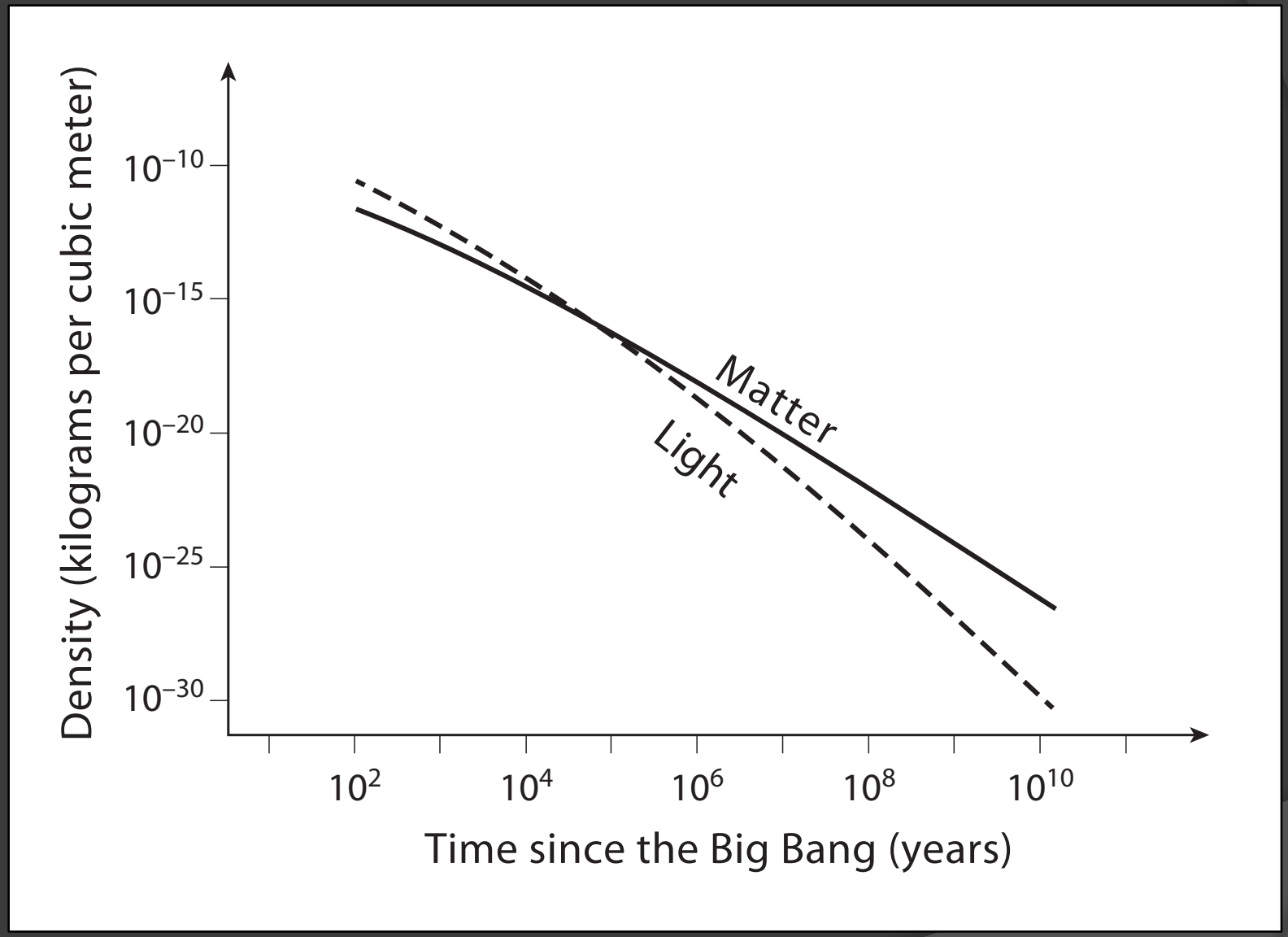


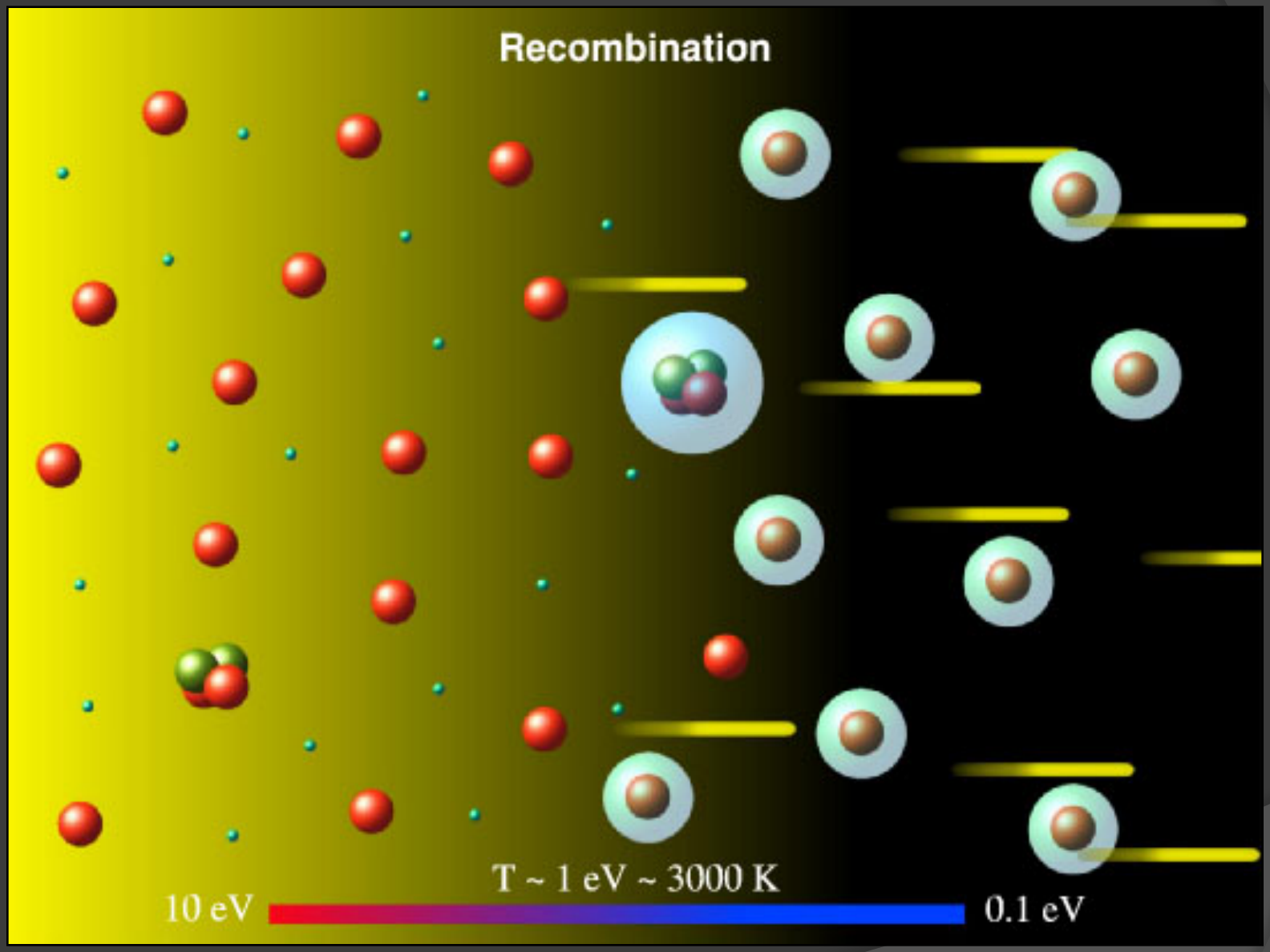

^1H
^1H
ν
^2H
^1H
γ
^3He
^1H
^4He
^1H
^1H
ν
^2H
^1H
γ
^3He
^1H

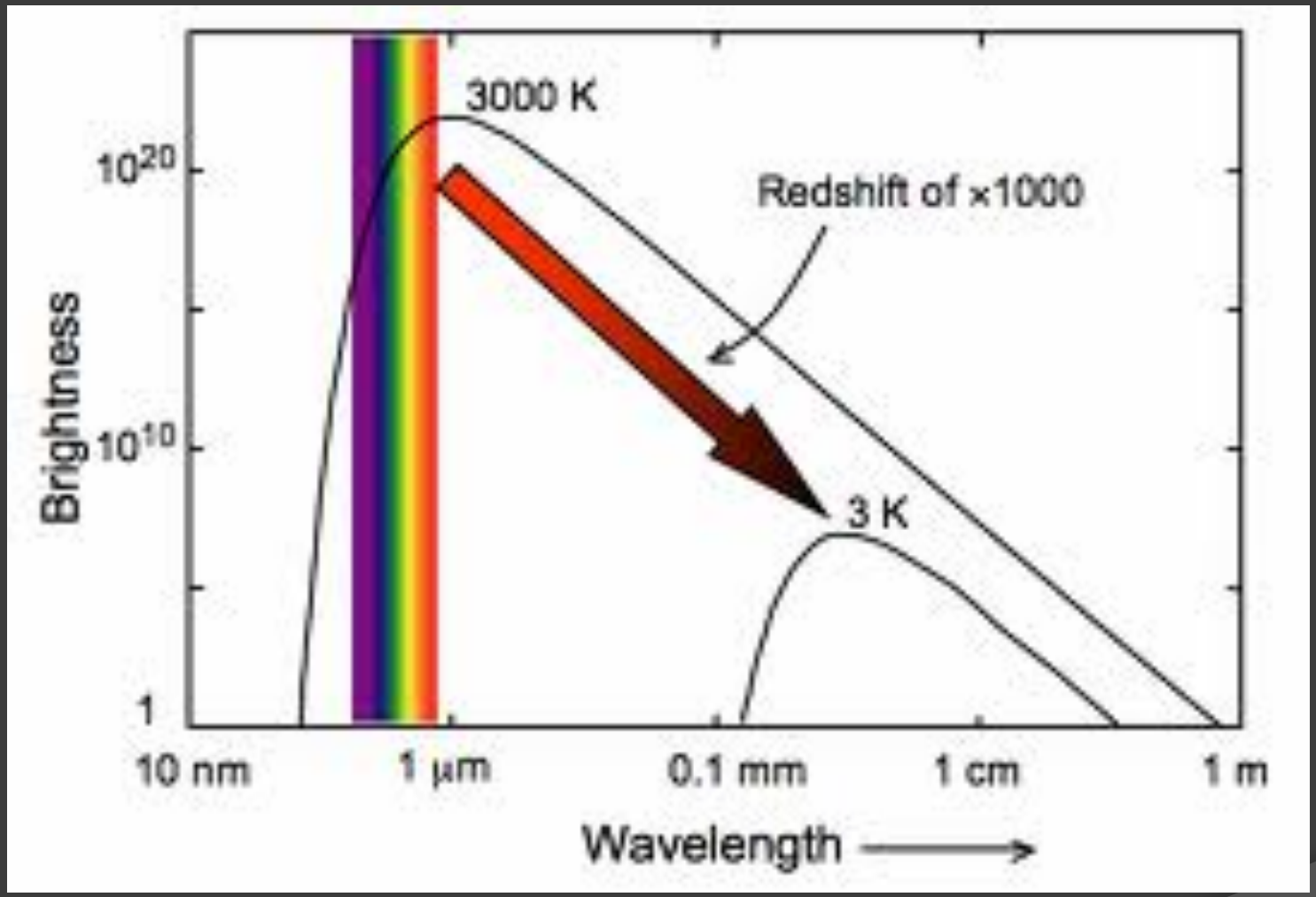

proton
p
deuterium
p
n
n
neutron
p
n
n
tritium
p
p
n
n
p
p
n
helium

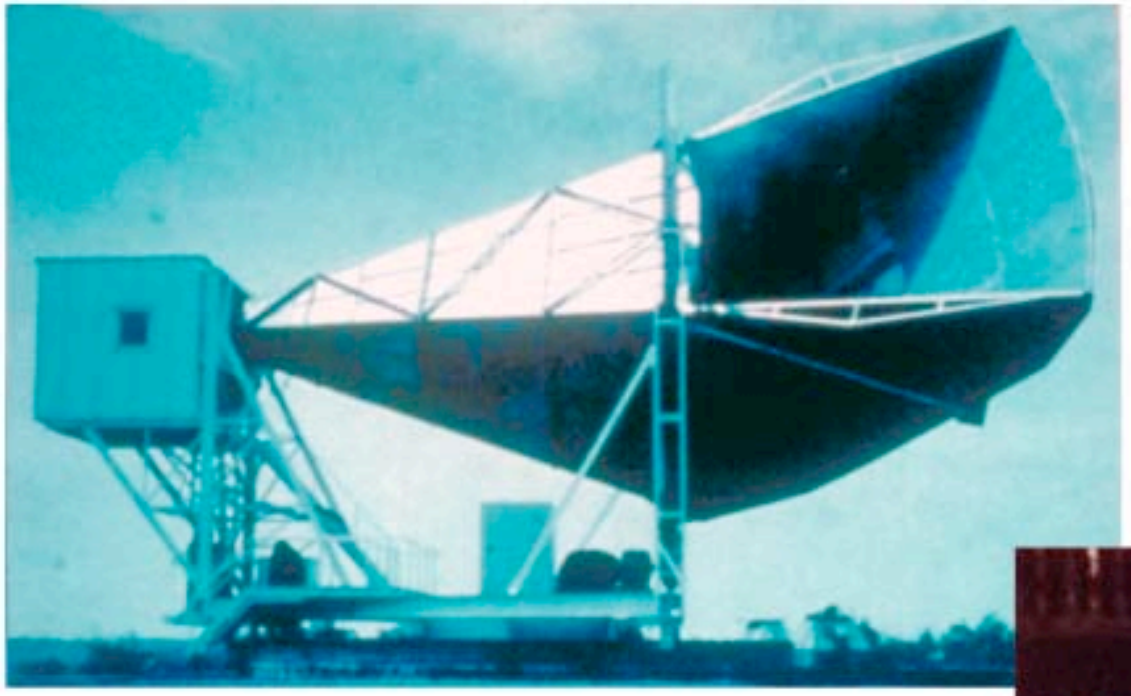

0
-5
-10
log(mass fraction)
0
1
2
3
4
log(t [sec])
H
^{4}He
D
^{3}He
^{3}H
n
^{7}Be
^{7}Li
^{6}Li


Abundance of Si
is normalized to 10^6
Log_{10}(Abundance)
Z, Atomic number
H
He
Li
Be
B
C
N
O
F
Ne
Na
Si
P
S
Ar
Ca
Sc
Ti
V
Fe
Co
Ni
Cu
Zn
Ga
Ge
As
Zr
Nb
Mo
Sn
In
Te
Xe
Ba
Pr
W
Re
Pt
Au
Hg
Pb
Bi
Th
U


Density (kilograms per cubic meter)
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
Matter
Light
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}
Time since the Big Bang (years)

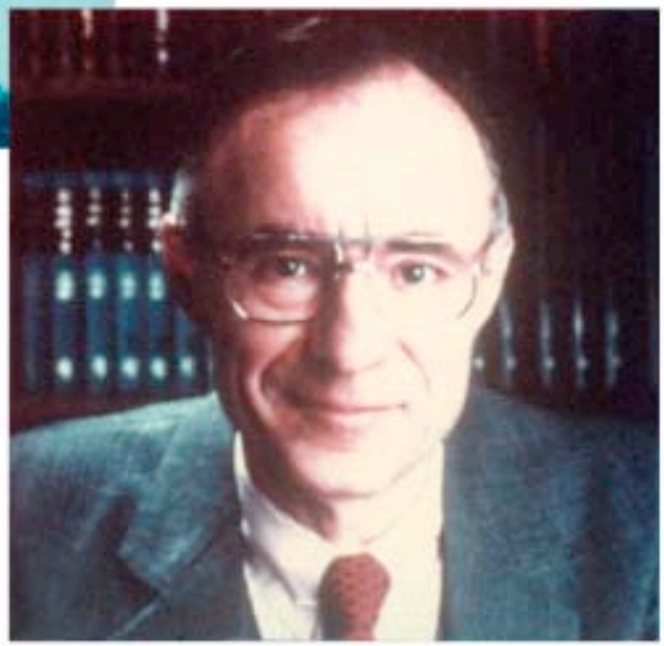

Recombination
T ~ 1 eV ~ 3000 K
10 eV
0.1 eV


3000 K
Redshift of ×1000
3 K
Brightness
10^{20}
10^{10}
1
10 nm
1 μm
0.1 mm
1 cm
1 m
Wavelength



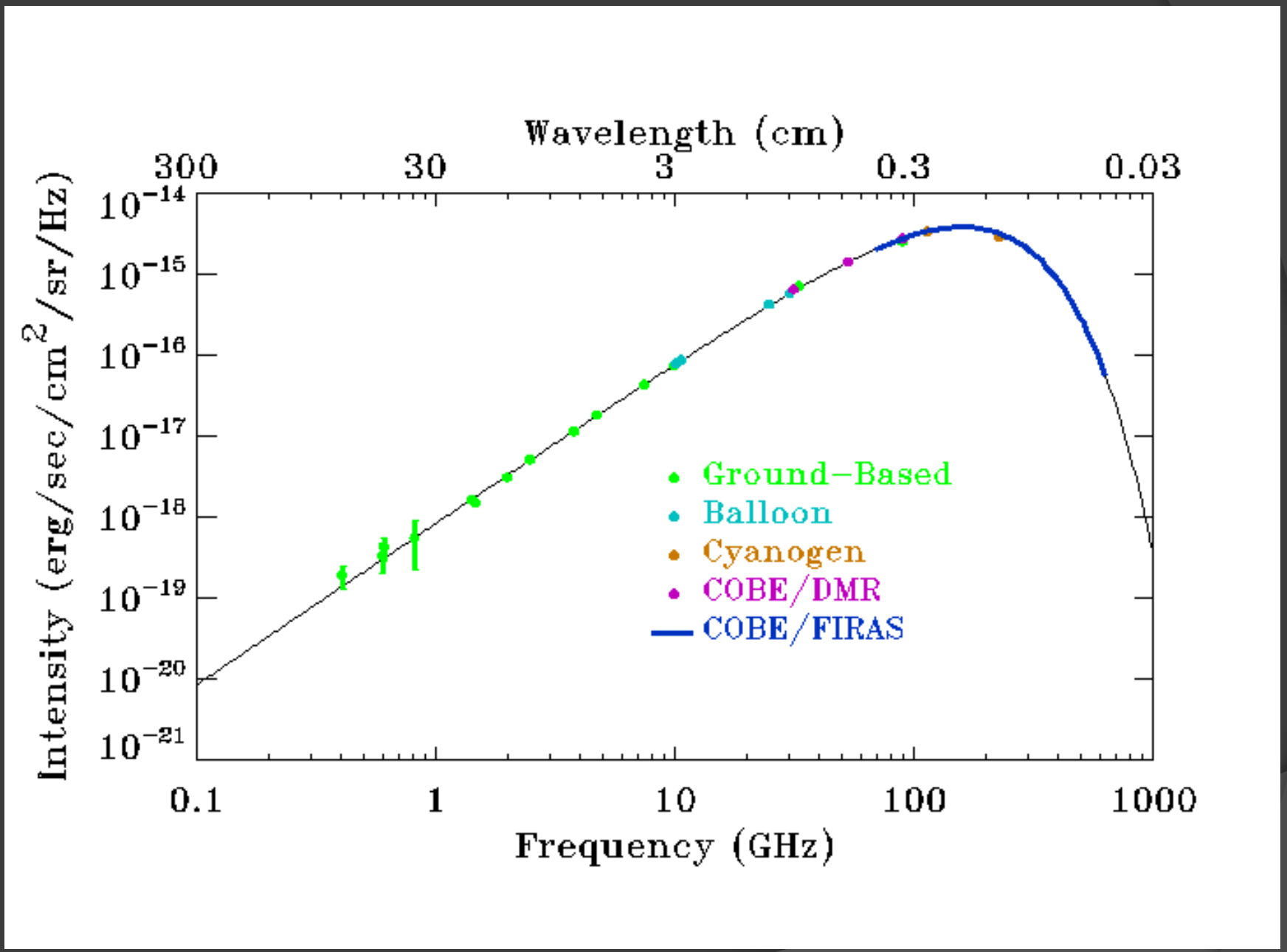
Microwave Receiver

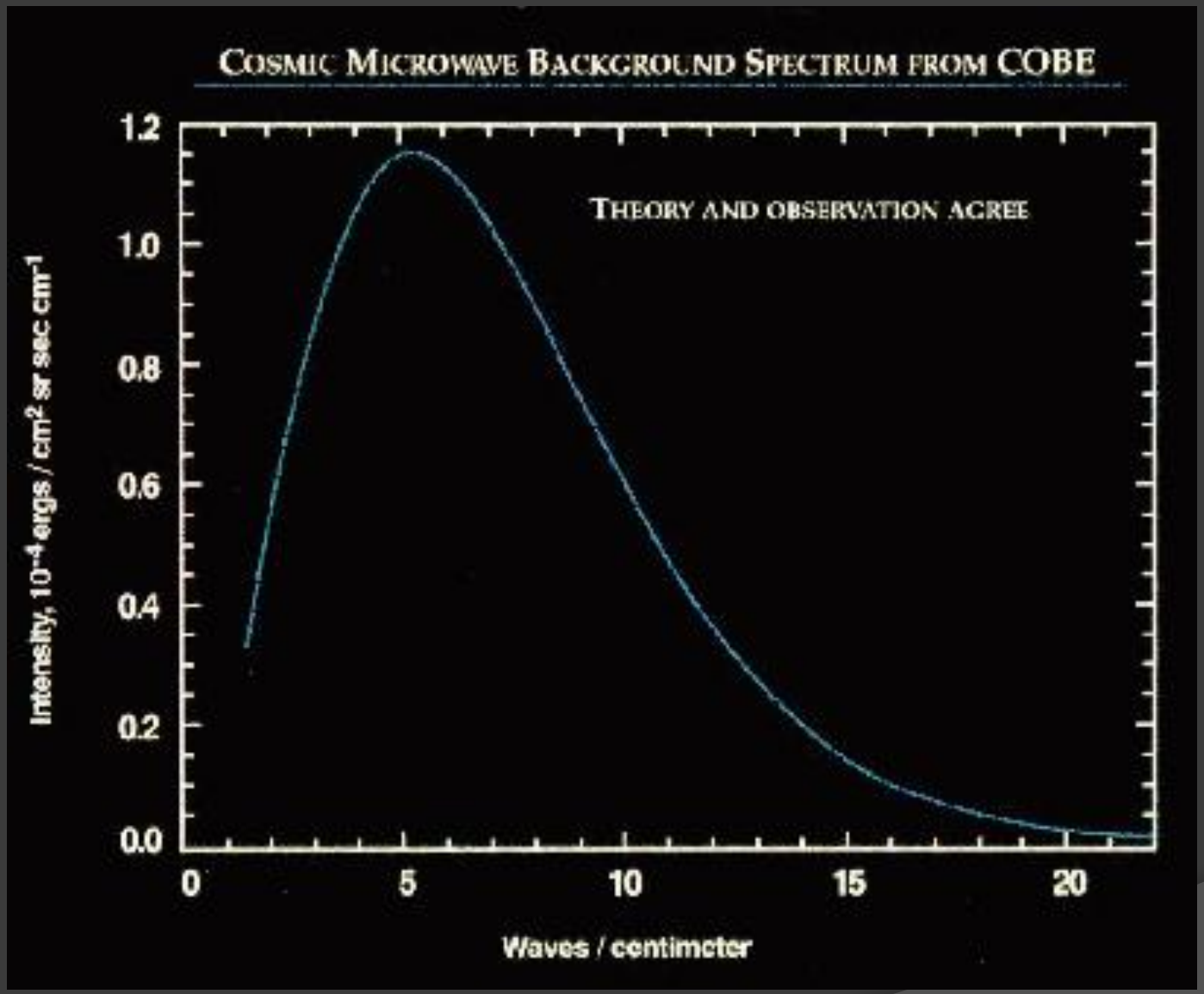
Robert Wilson



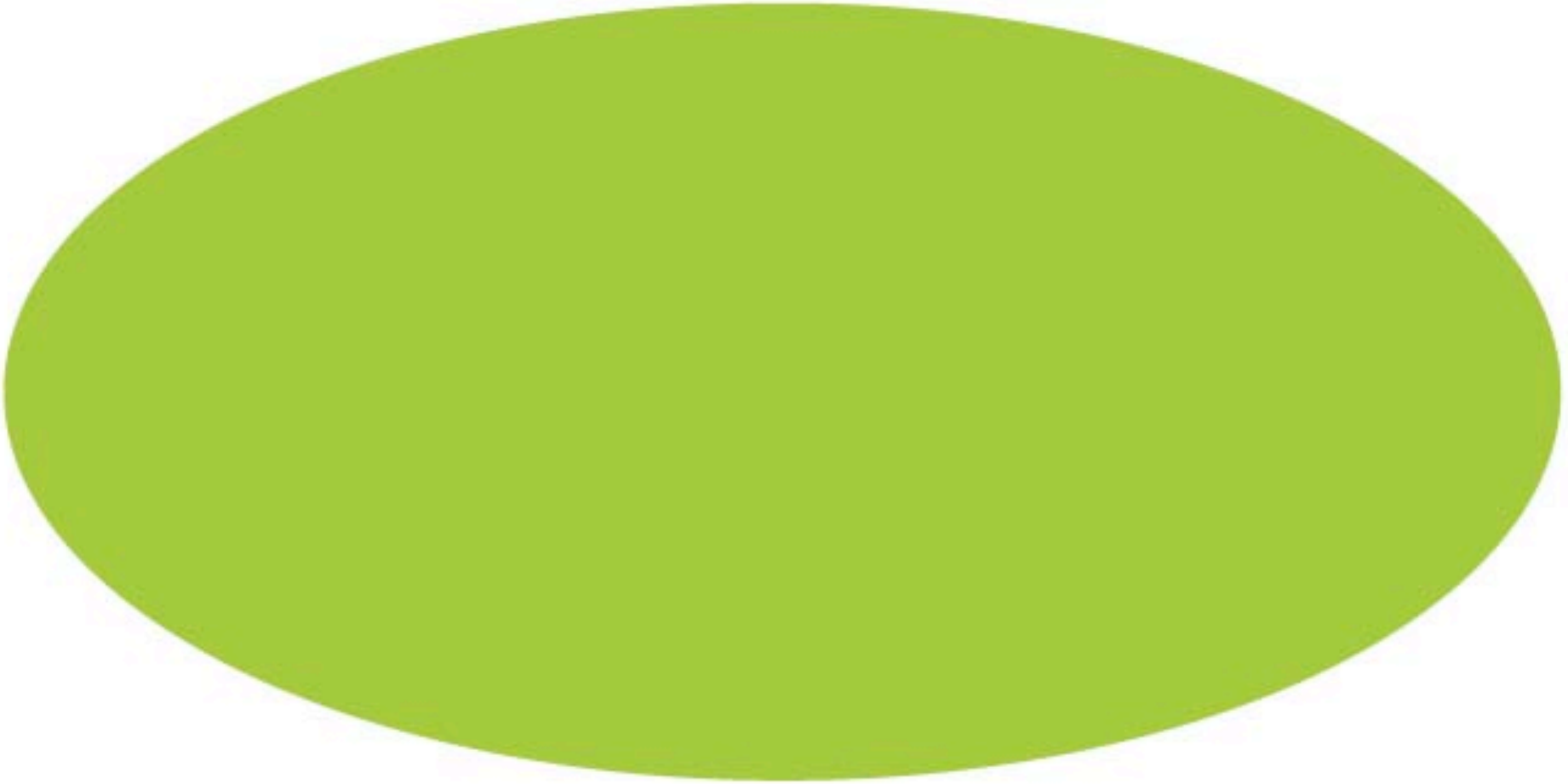
Arno Penzias

MAP990045

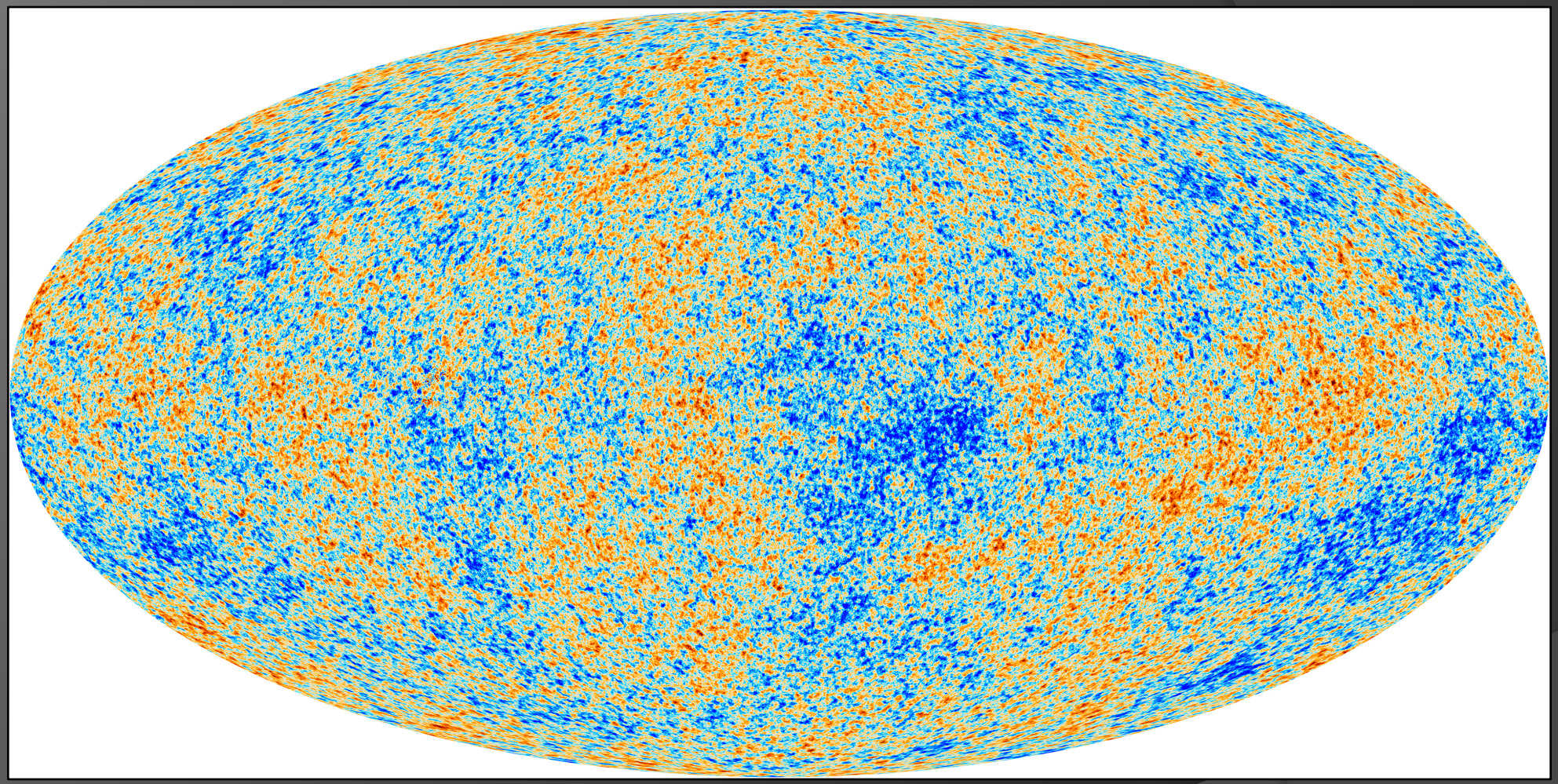

Wavelength (cm)
300
30
3
0.3
0.03
Intensity (erg/sec/cm^2/sr/Hz)
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
0.1
1
10
100
1000
Frequency (GHz)
Ground-Based
Balloon
Cyanogen
COBE/DMR
COBE/FIRAS

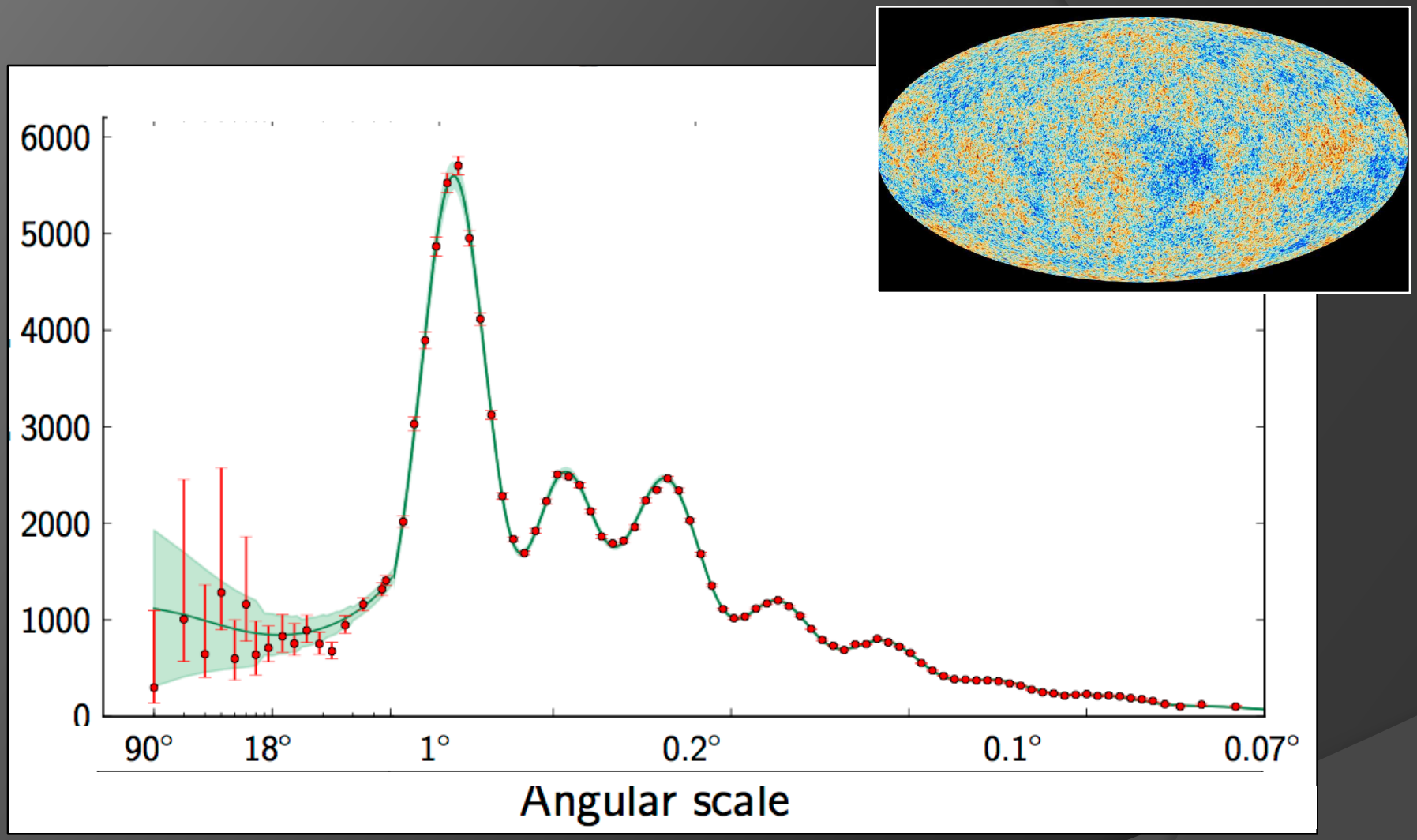

COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE
THEORY AND OBSERVATION AGREE
Intensity, 10^{-4} ergs / cm^2 sr sec cm^{-1}
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Waves / centimeter

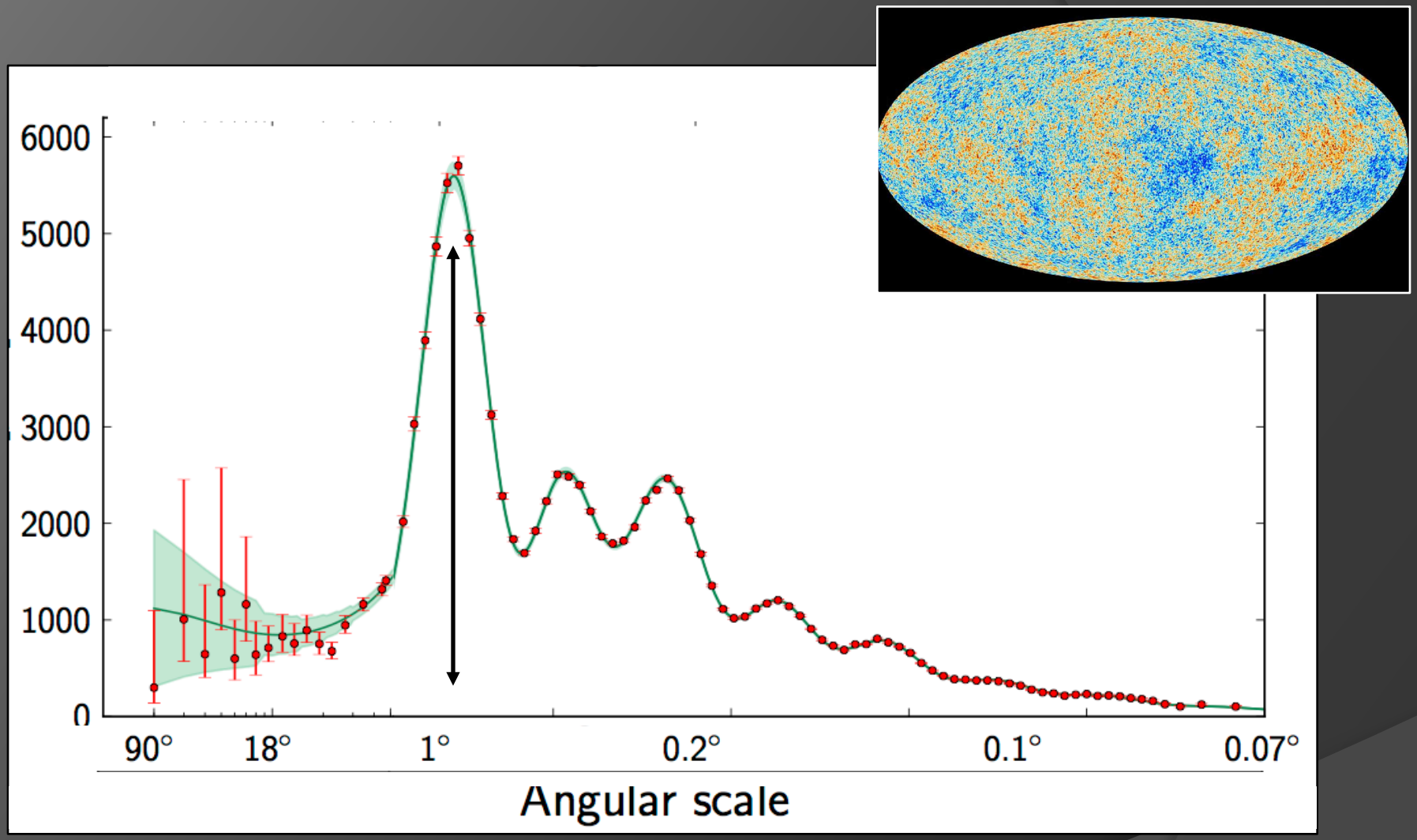
Isotropy of the Cosmic Microwave Background

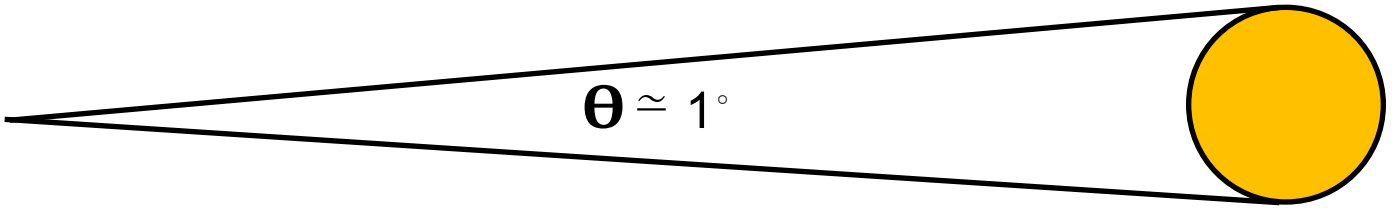


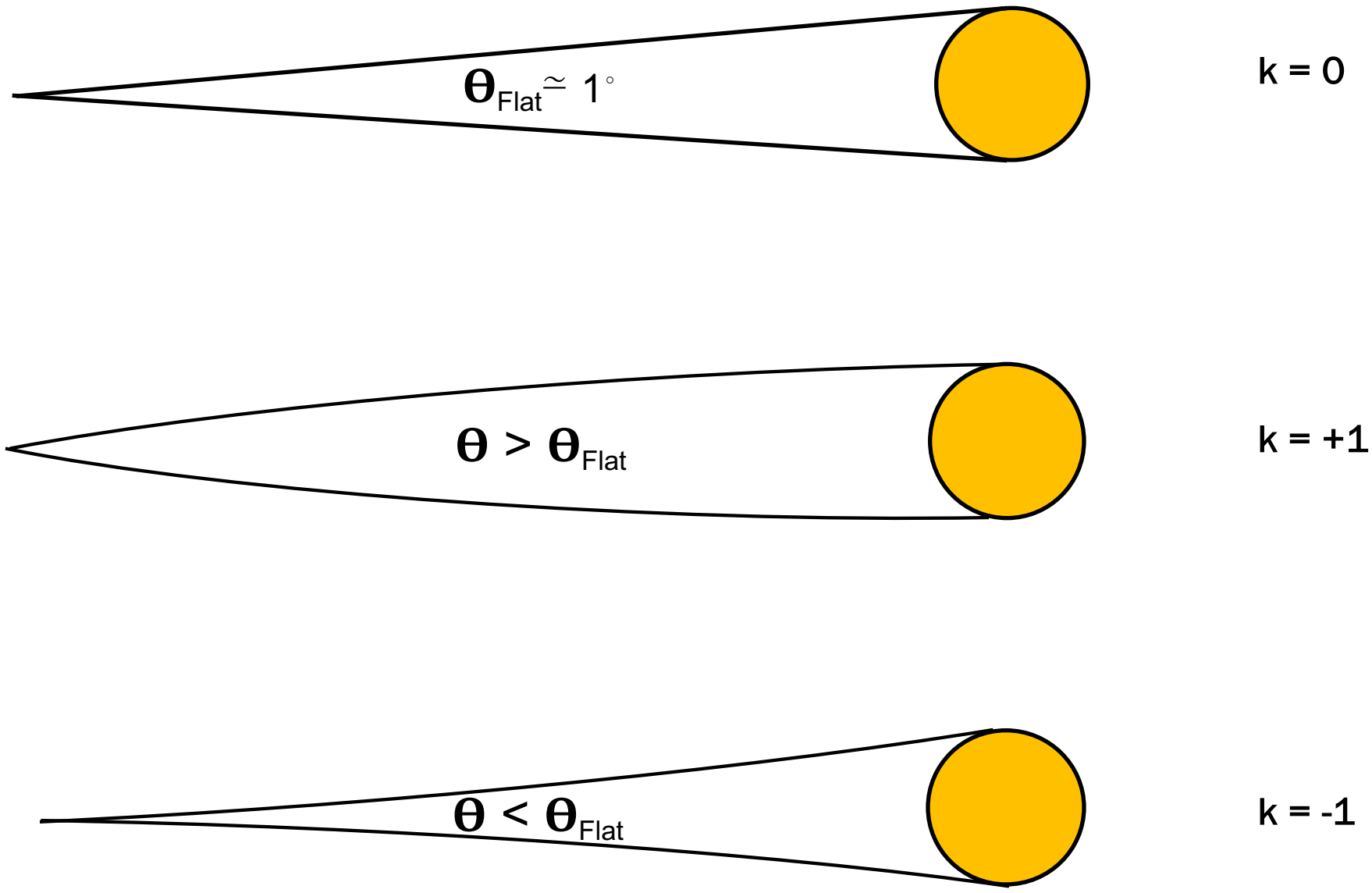
MAP990004

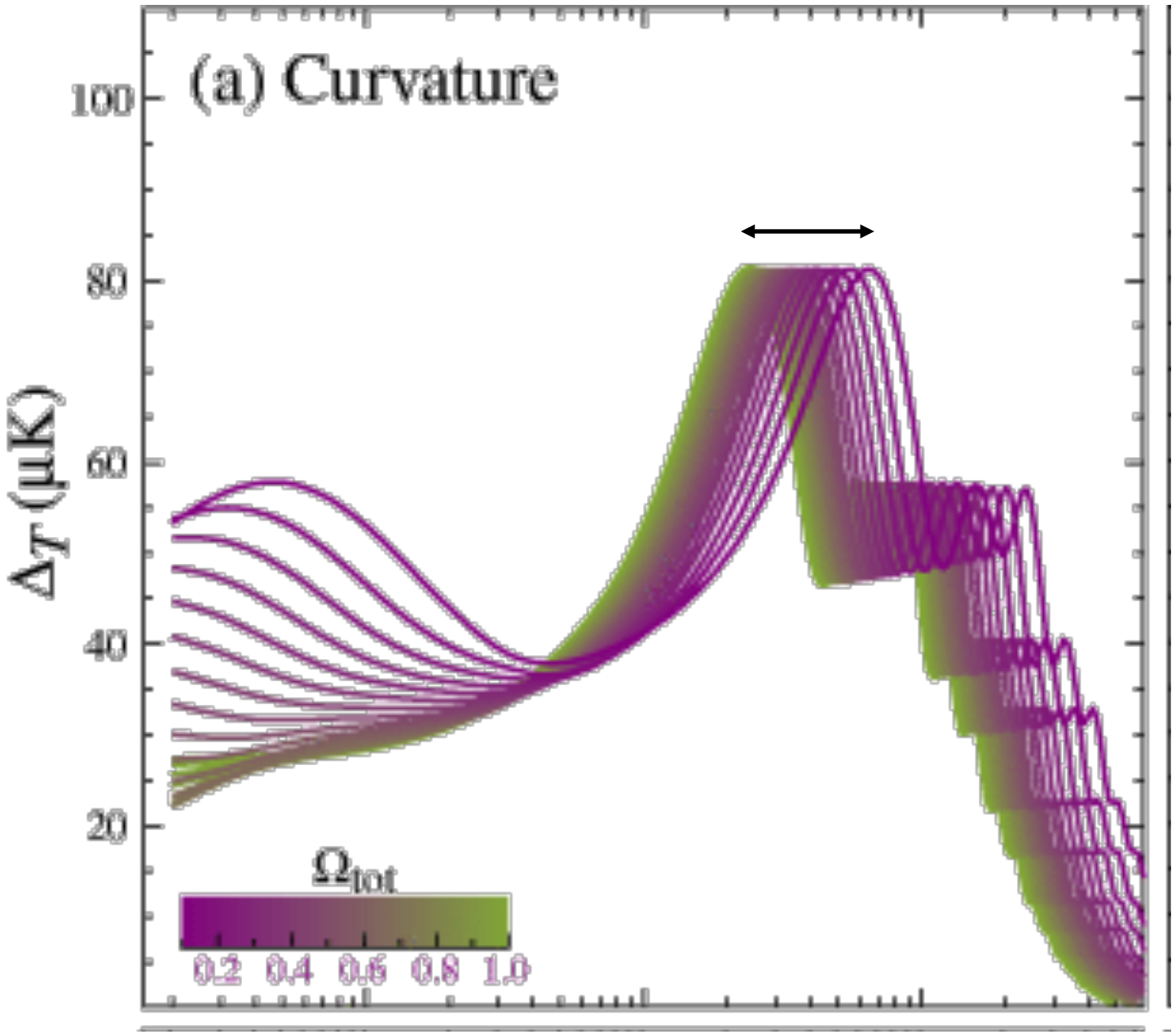


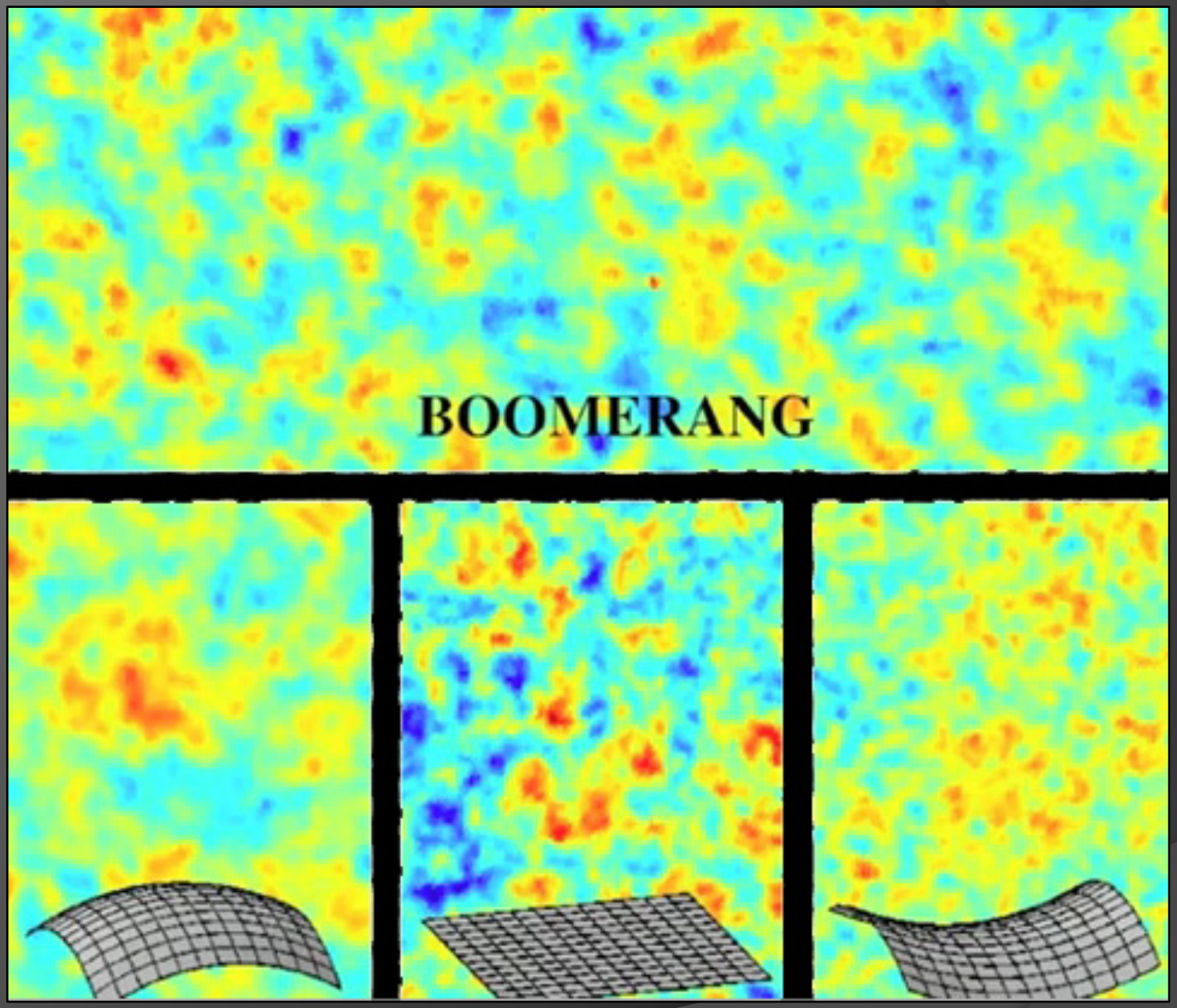

6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale

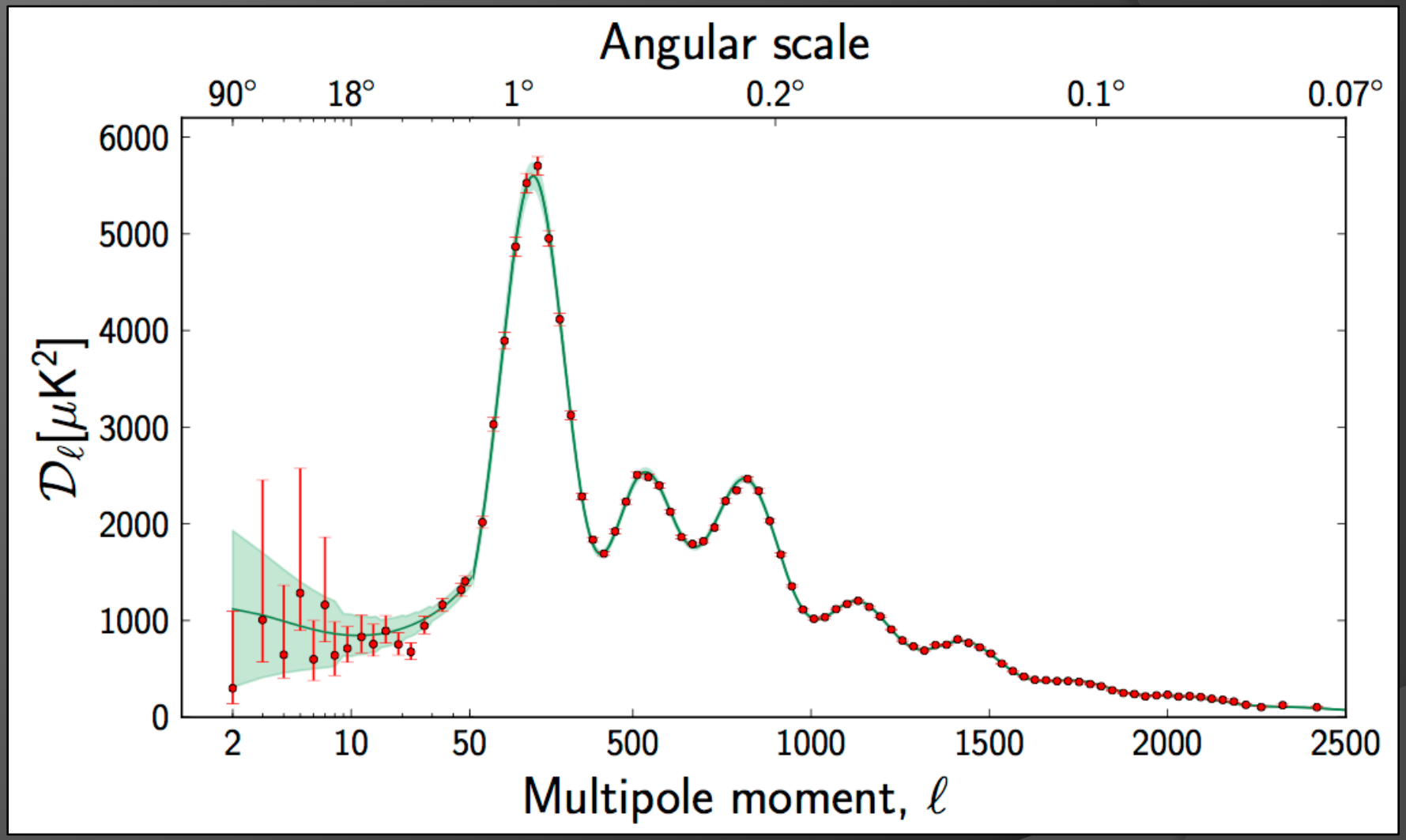

6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale

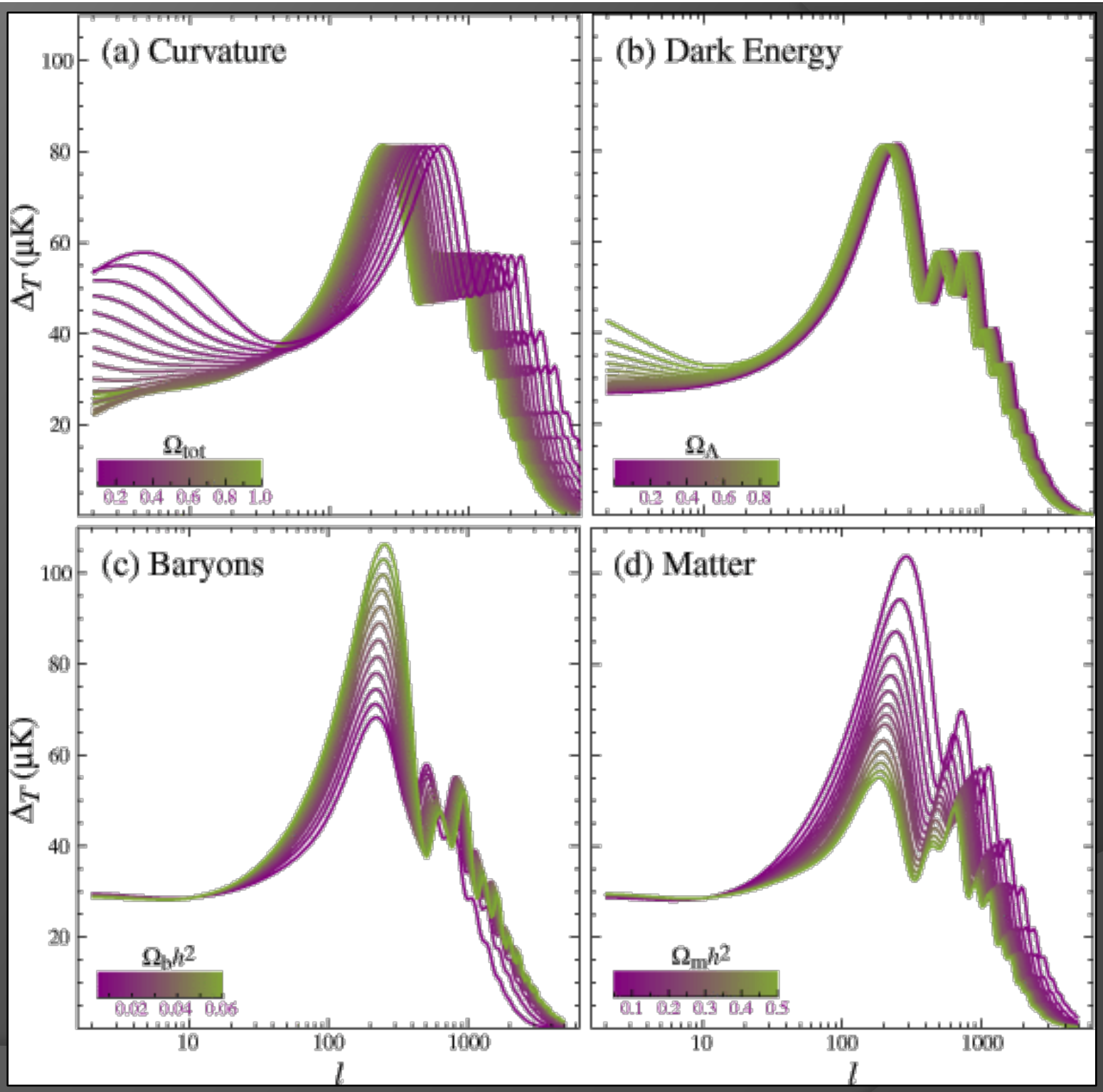

$\theta \simeq 1^\circ$

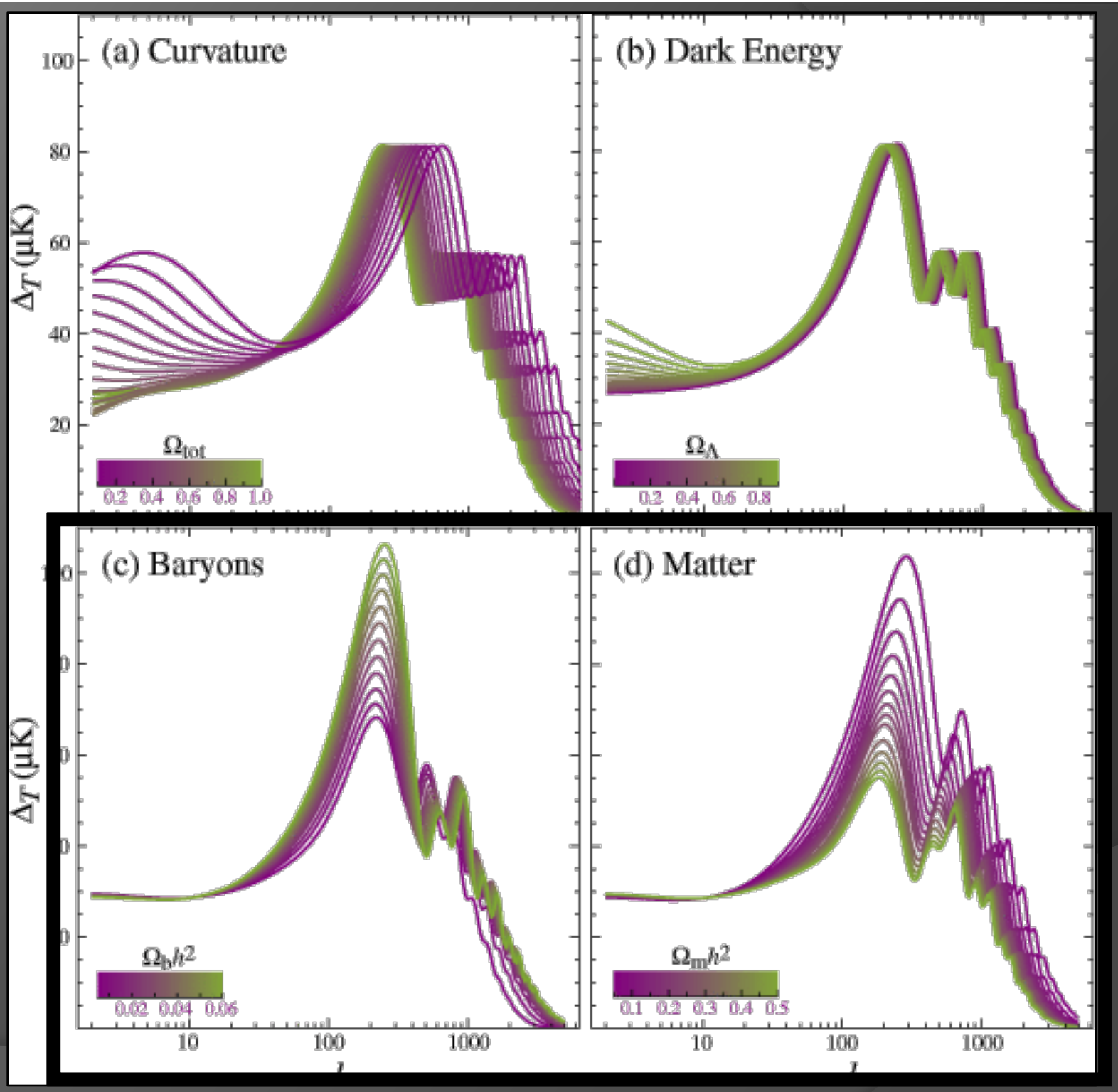

$\theta_{Flat} \simeq 1^\circ$
k = 0
$\theta > \theta_{Flat}$
k = +1
$\theta < \theta_{Flat}$
k = -1

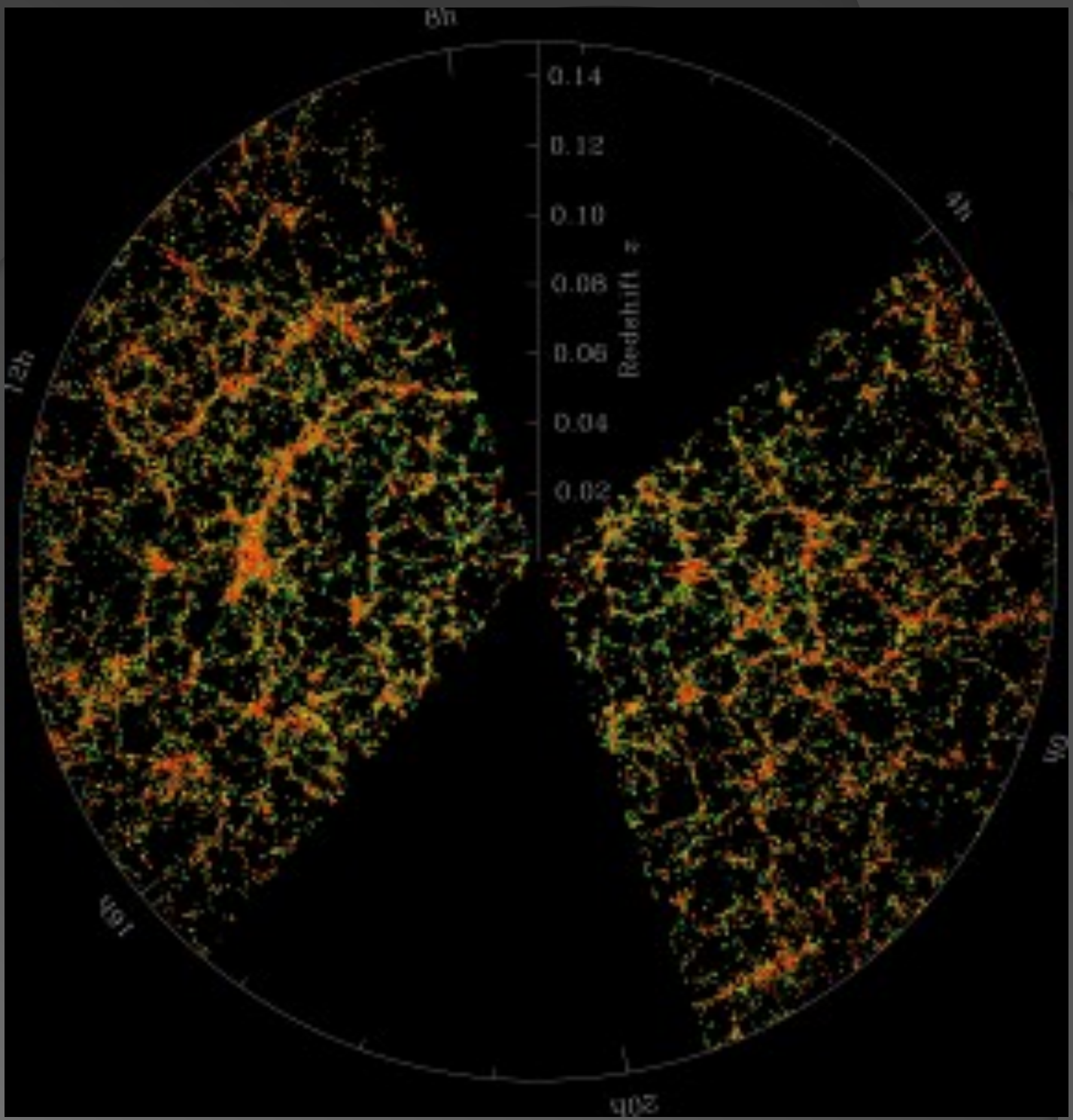

(a) Curvature
100
80
60
40
20
Δ_T (μK)
Ω_{tot}
0.2 0.4 0.6 0.8 1.0

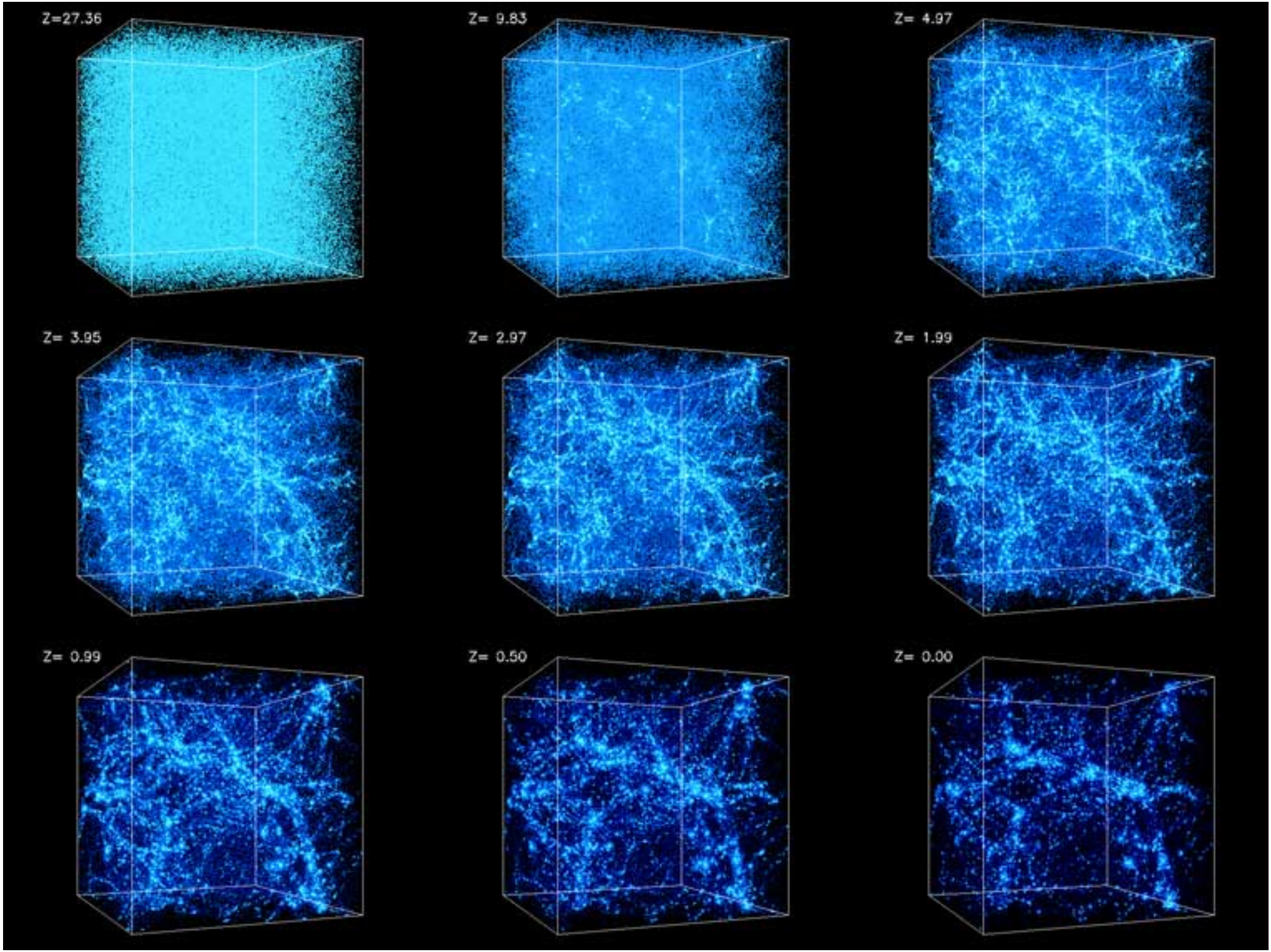

BOOMERANG

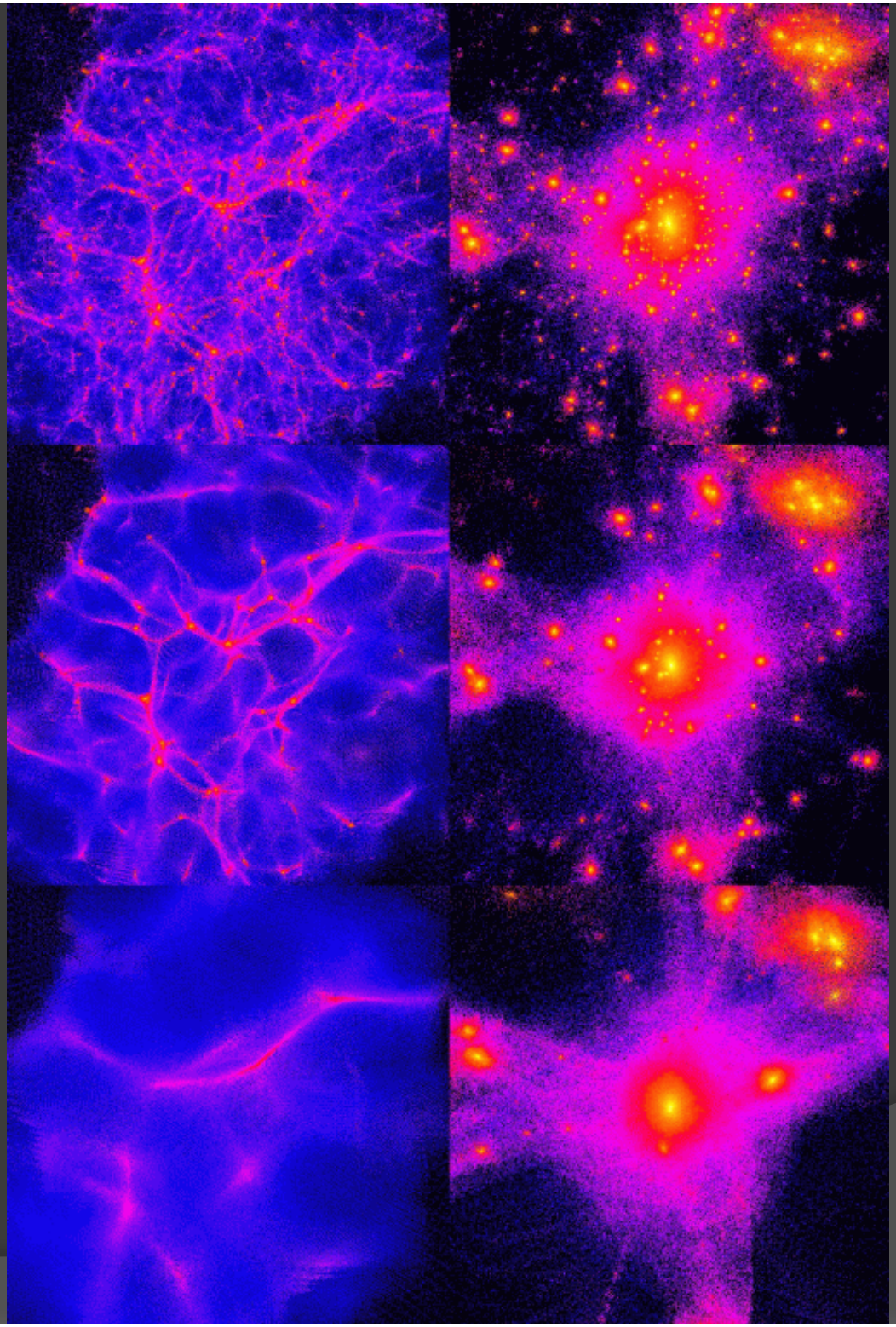

Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell[\mu K^2]$
2
10
50
500
1000
1500
2000
2500
Multipole moment, ℓ

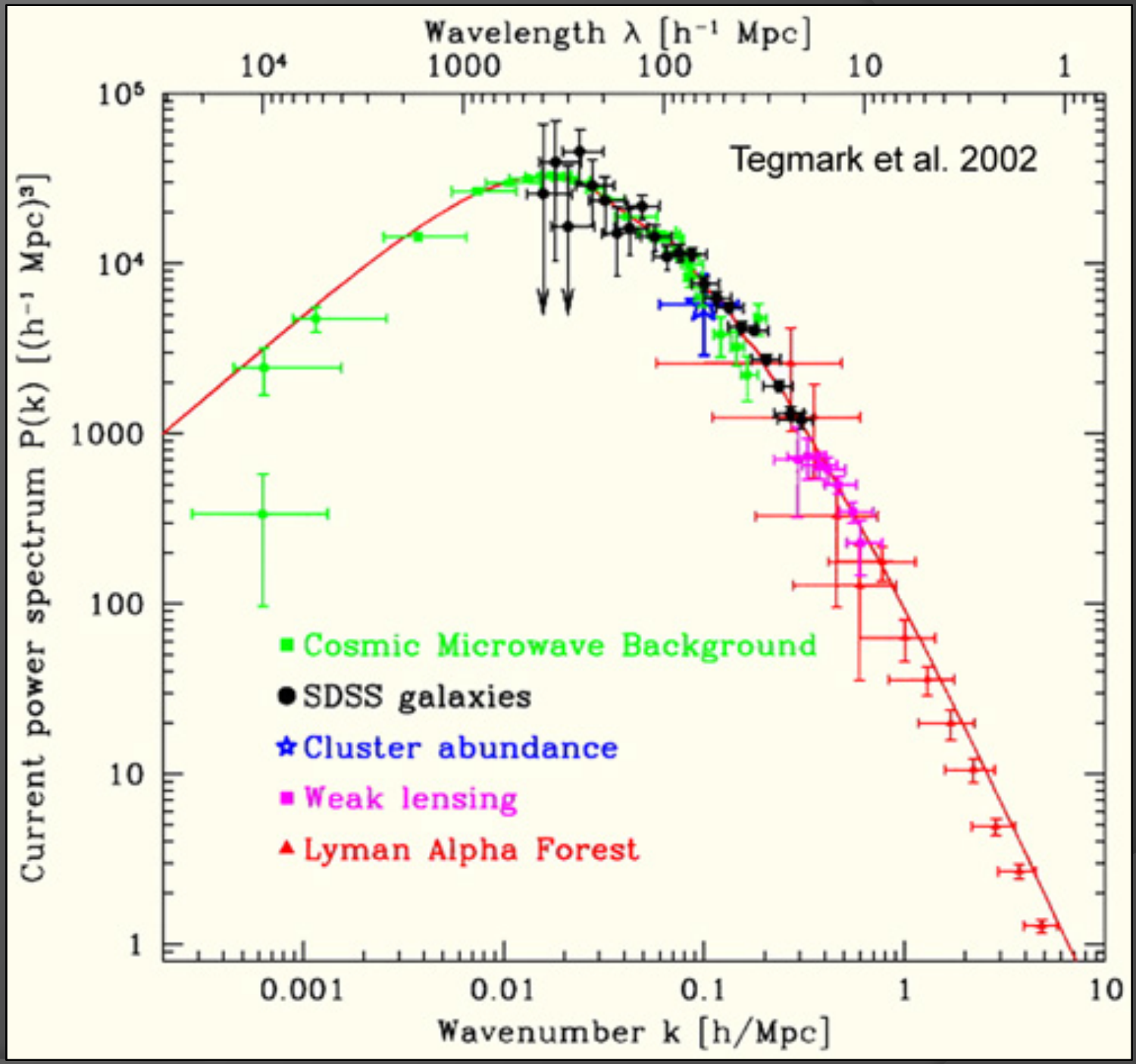

(a) Curvature
(b) Dark Energy
(c) Baryons
(d) Matter
Δ_T (μK)
l
Ω_{tot}
0.2 0.4 0.6 0.8 1.0
Ω_Λ
0.2 0.4 0.6 0.8
$\Omega_b h^2$
0.02 0.04 0.06
$\Omega_m h^2$
0.1 0.2 0.3 0.4 0.5
20
40
60
80
100
10
100
1000

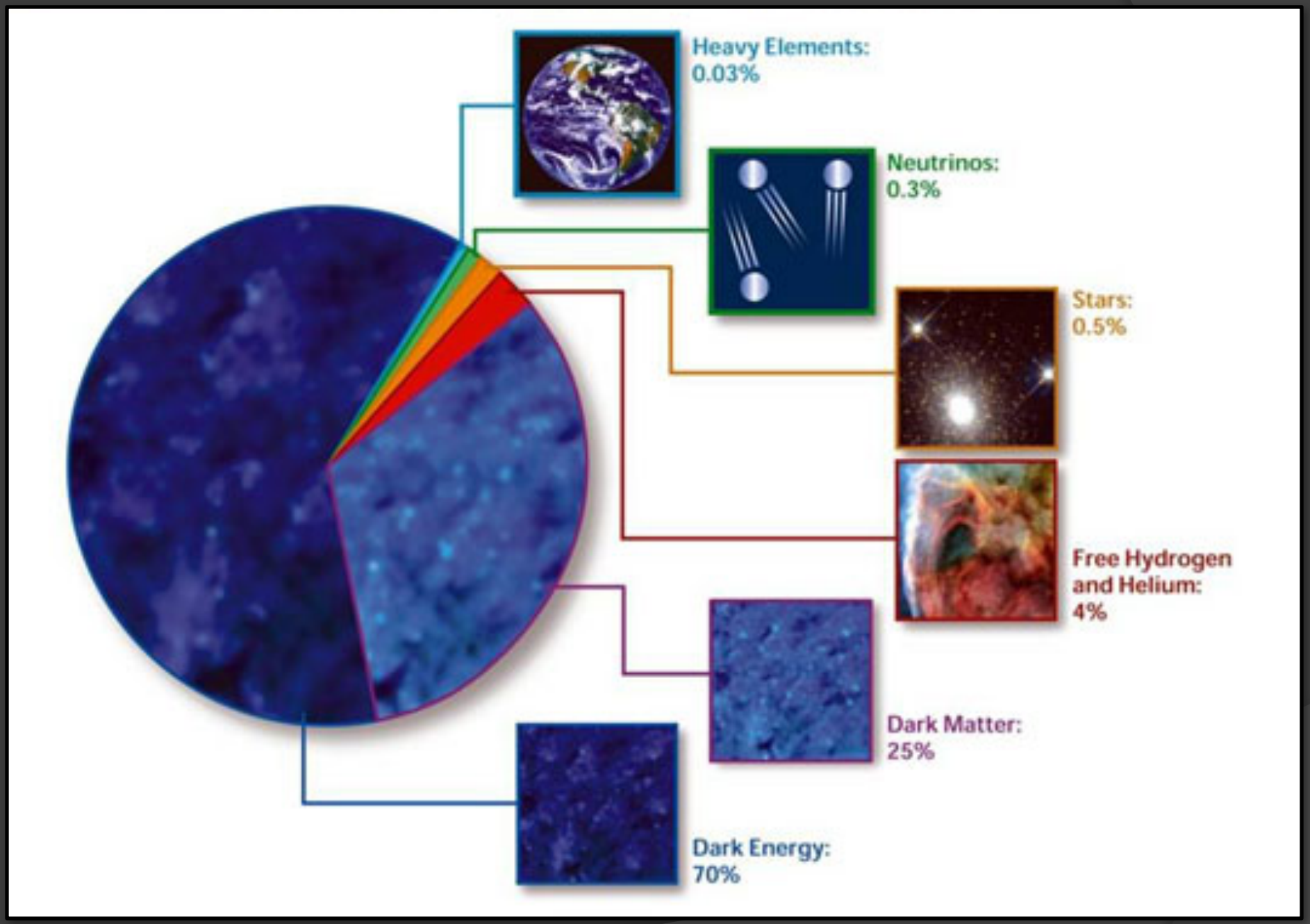

(a) Curvature
(b) Dark Energy
(c) Baryons
(d) Matter
Δ_T (μK)
100
80
60
40
20
Ω_{tot}
0.2 0.4 0.6 0.8 1.0
Ω_Λ
0.2 0.4 0.6 0.8
$\Omega_b h^2$
0.02 0.04 0.06
$\Omega_m h^2$
0.1 0.2 0.3 0.4 0.5
10
100
1000
l

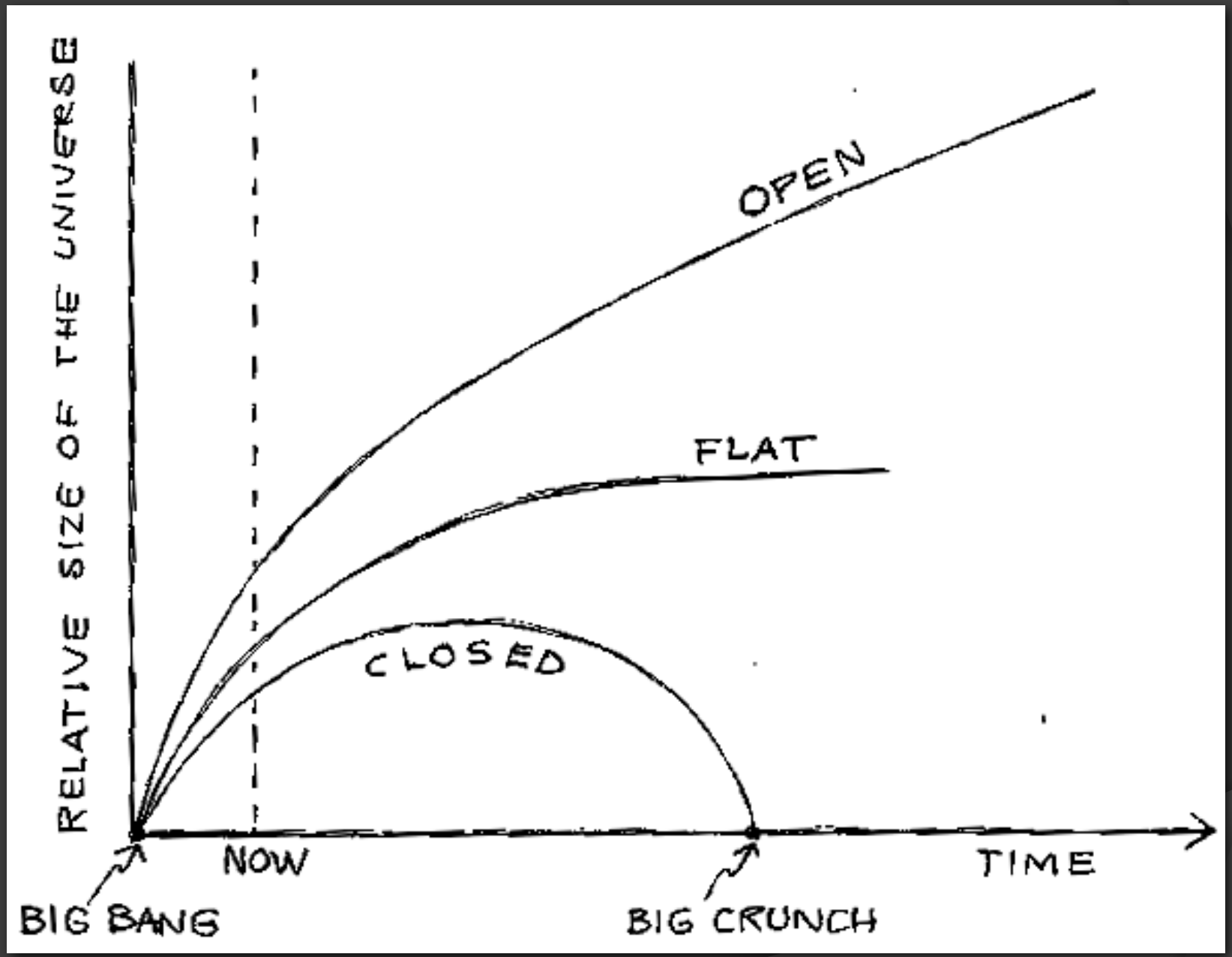

8h
0.14
0.12
0.10
0.08
0.06
0.04
0.02
Redshift z
4h
12h
16h
20h

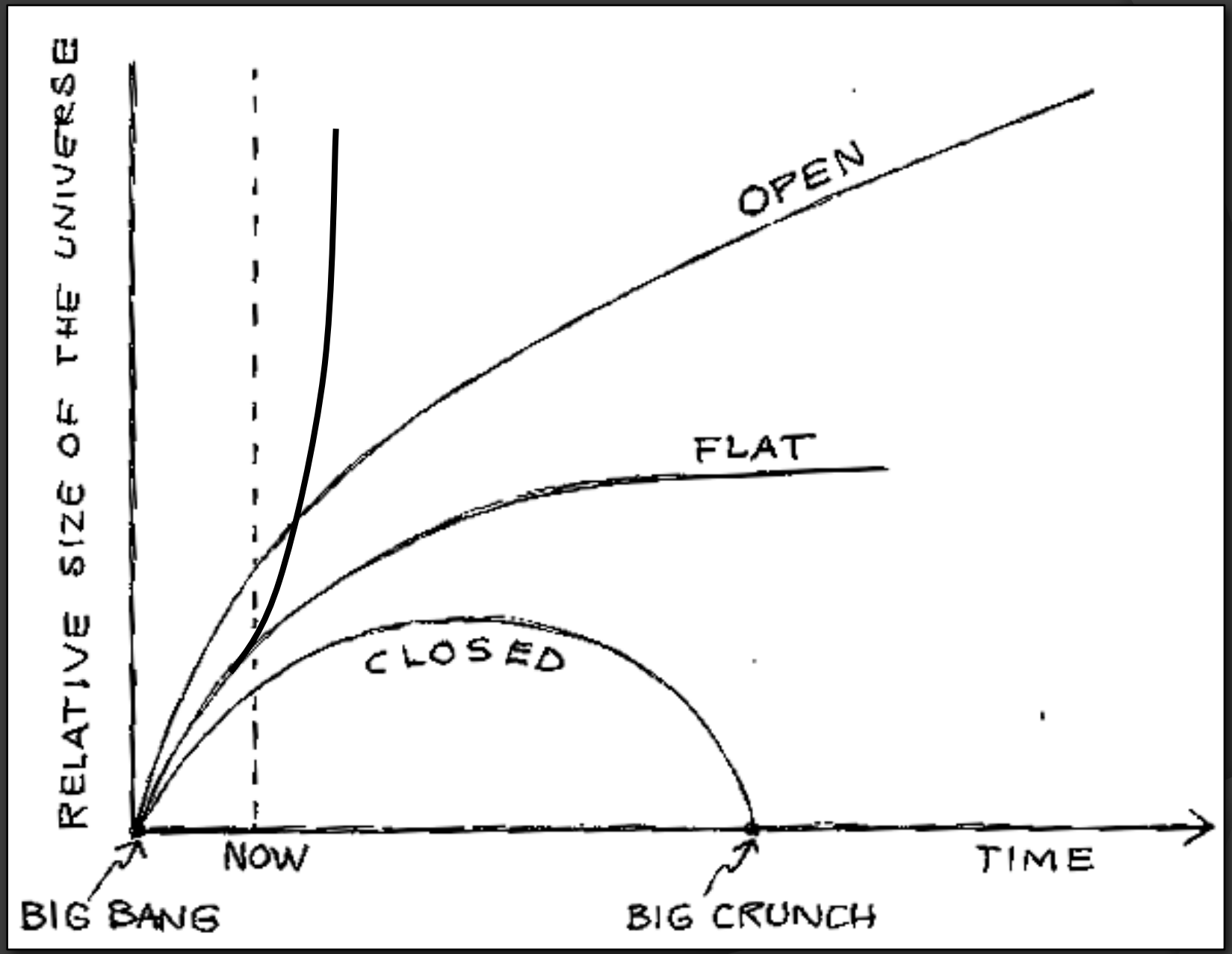

Z=27.36
Z= 9.83
Z= 4.97
Z= 3.95
Z= 2.97
Z= 1.99
Z= 0.99
Z= 0.50
Z= 0.00

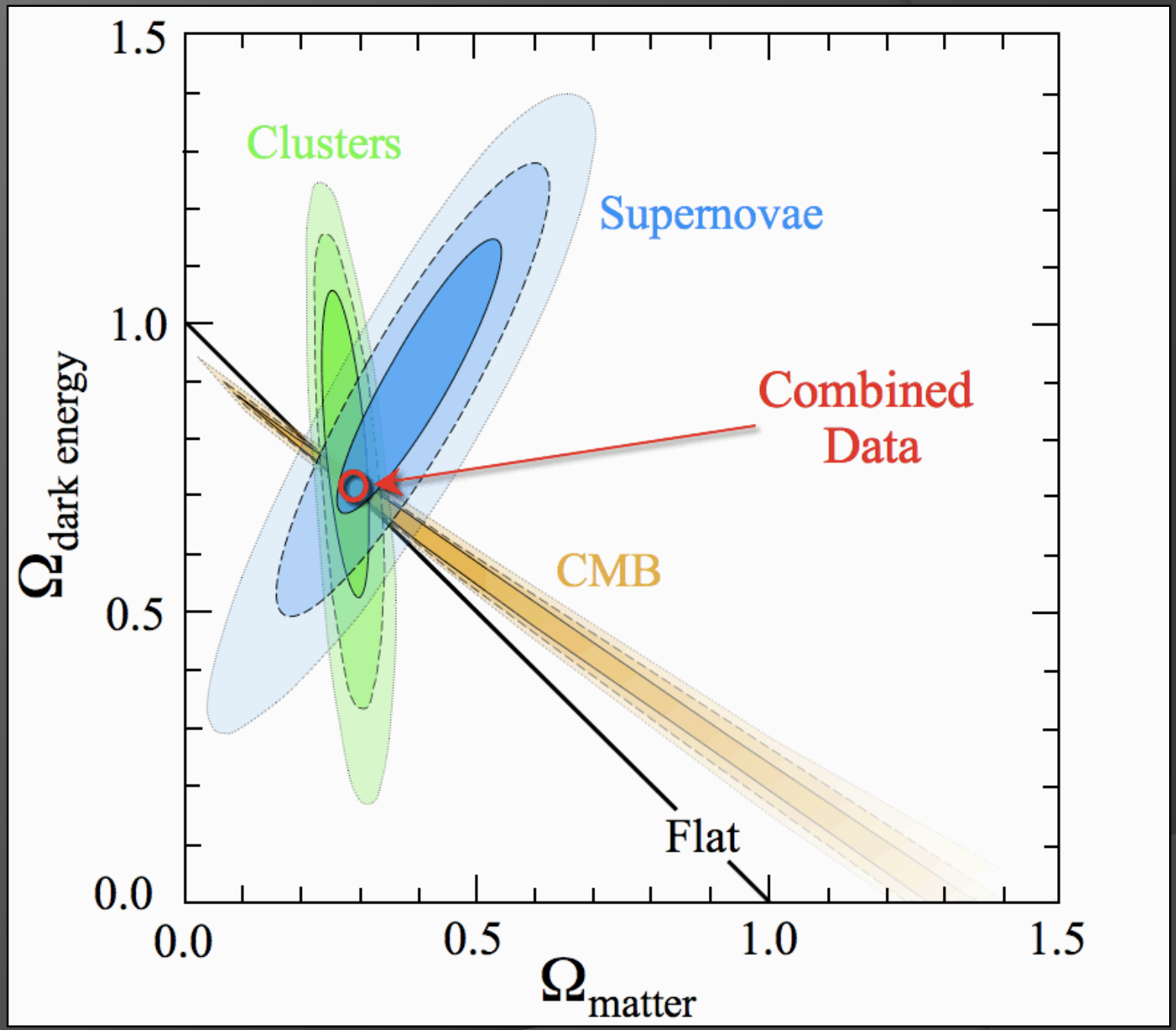



Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
Tegmark et al. 2002
Current power spectrum P(k) [(h⁻¹ Mpc)³]
10⁵
10⁴
1000
100
10
1
Cosmic Microwave Background
SDSS galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]


Heavy Elements:
0.03%
Neutrinos:
0.3%
Stars:
0.5%
Free Hydrogen
and Helium:
4%
Dark Matter:
25%
Dark Energy:
70%

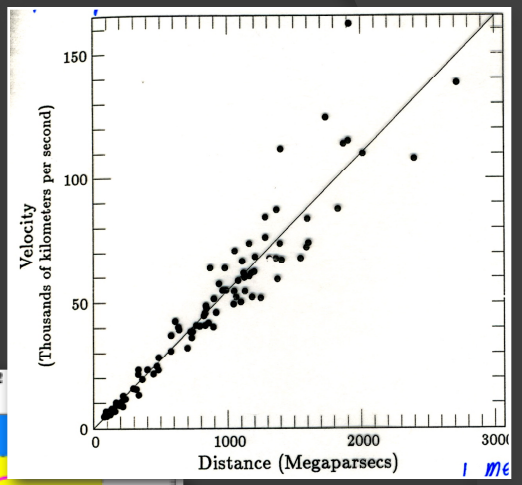

RELATIVE SIZE OF THE UNIVERSE
OPEN
FLAT
CLOSED
NOW
TIME
BIG BANG
BIG CRUNCH

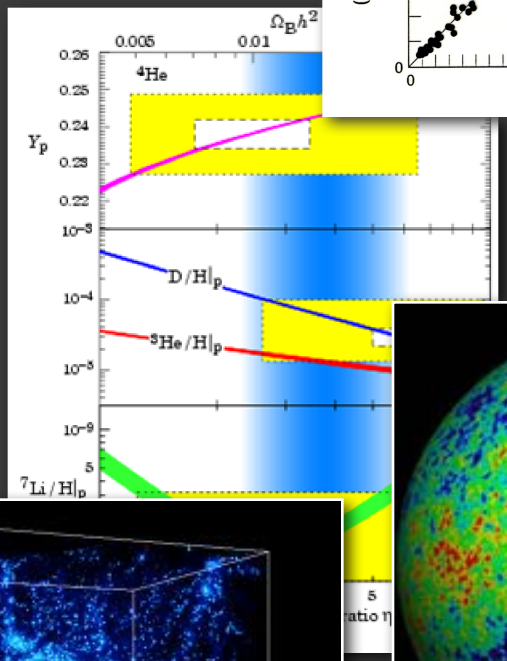

RELATIVE SIZE OF THE UNIVERSE
OPEN
FLAT
CLOSED
NOW
TIME
BIG BANG
BIG CRUNCH

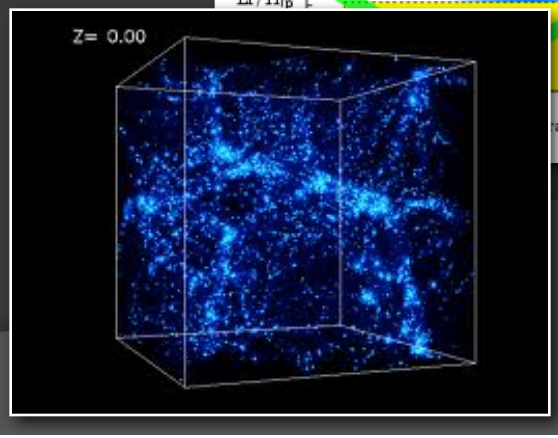

1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
Clusters
Supernovae
Combined
Data
CMB
Flat
$\Omega_{\text{dark energy}}$
Ω_{matter}

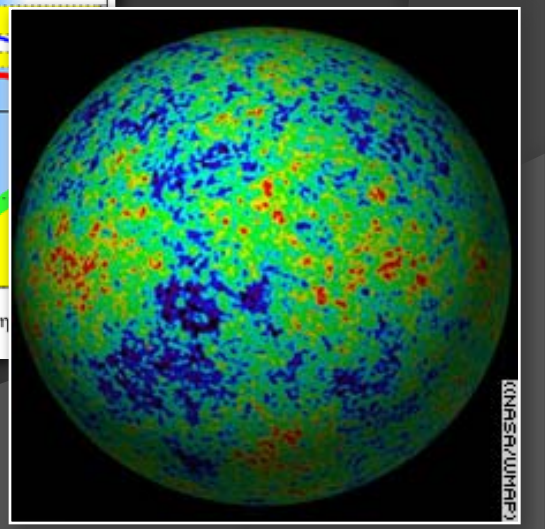
The Pillars Of Physical Cosmology

- Hubble Expansion
- The Light Element Abundances
- The Cosmic Microwave Background
- Large Scale Structure









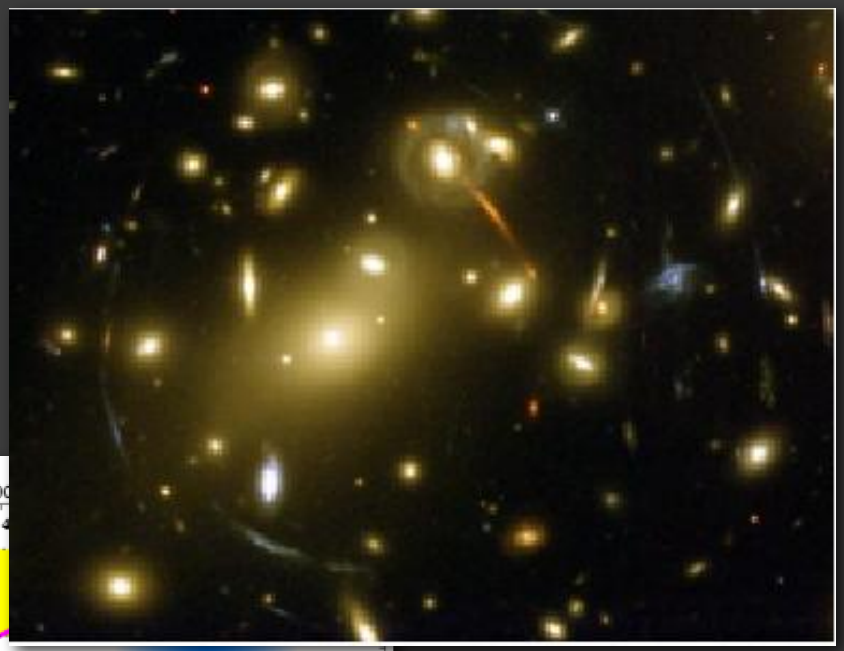
The Big Bang Theory, although remarkably successful, requires us to introduce:

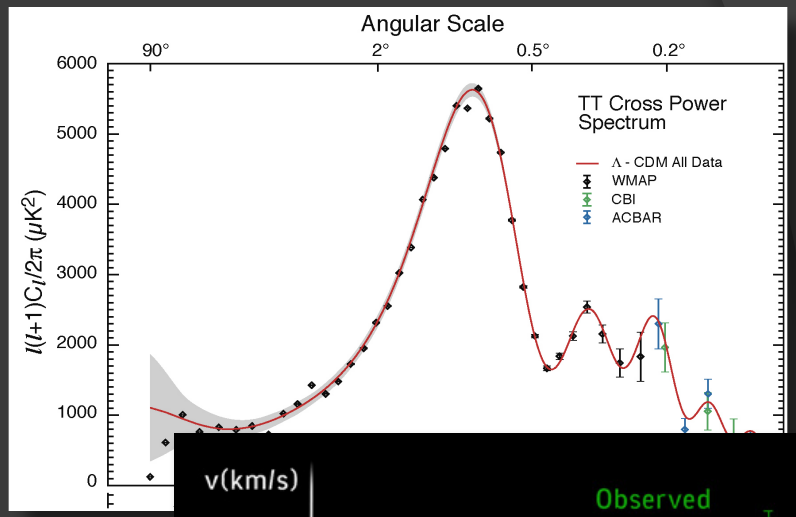
1) something that drives accelerated expansion in the early universe (inflation)

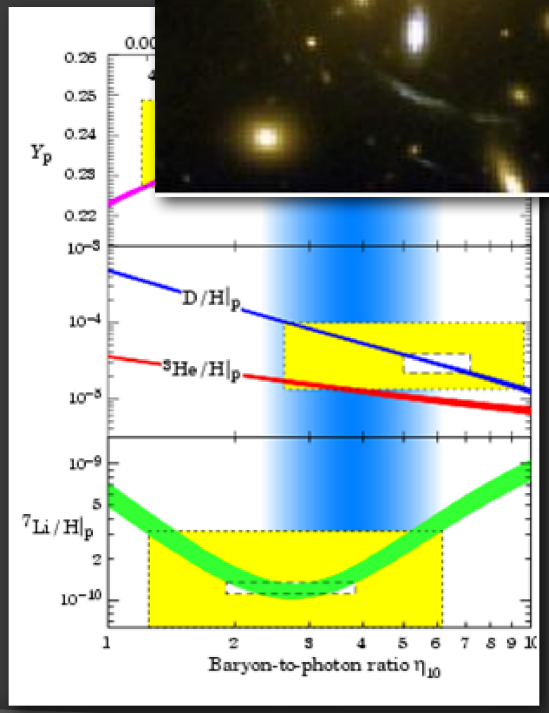
2) non-baryonic dark matter

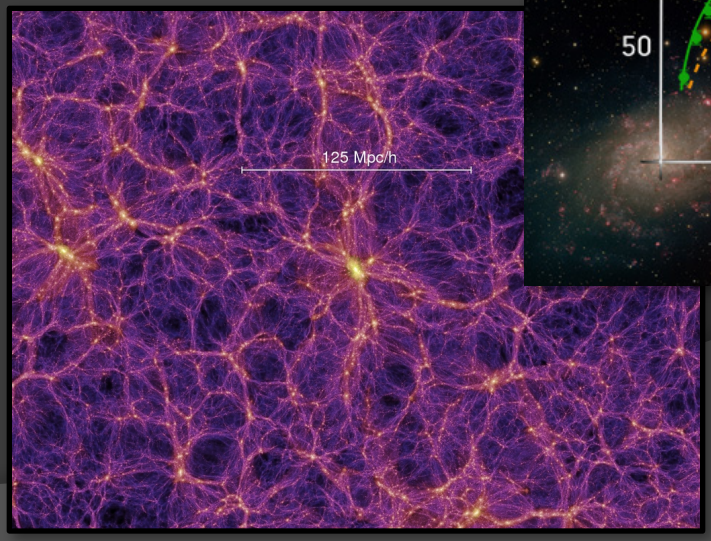
3) something that drives accelerated expansion in the current era (dark energy)

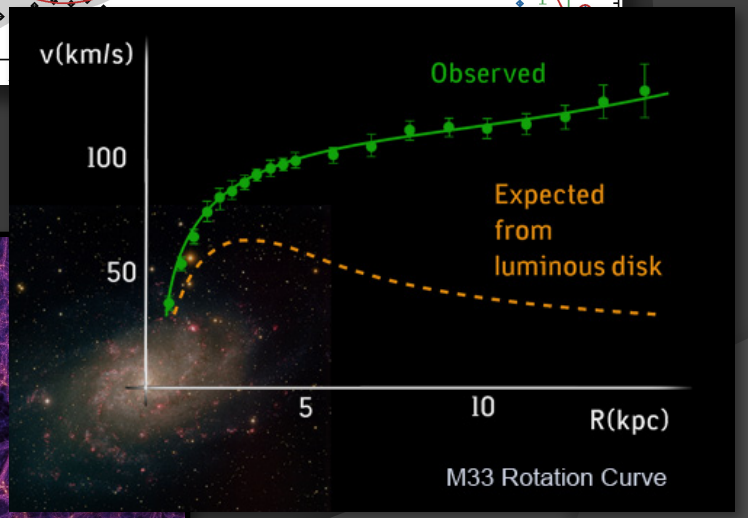
Evidence for Dark Matter







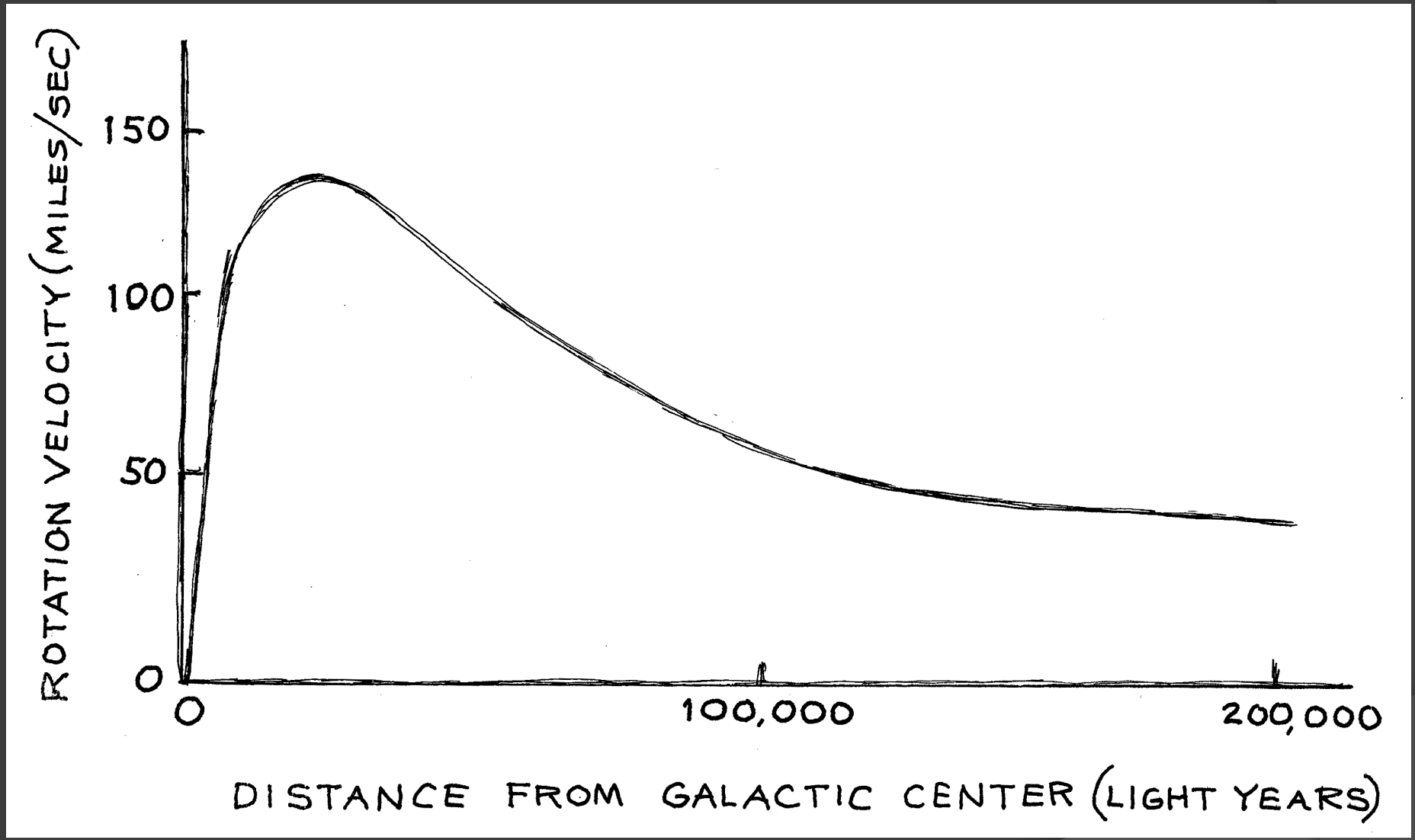


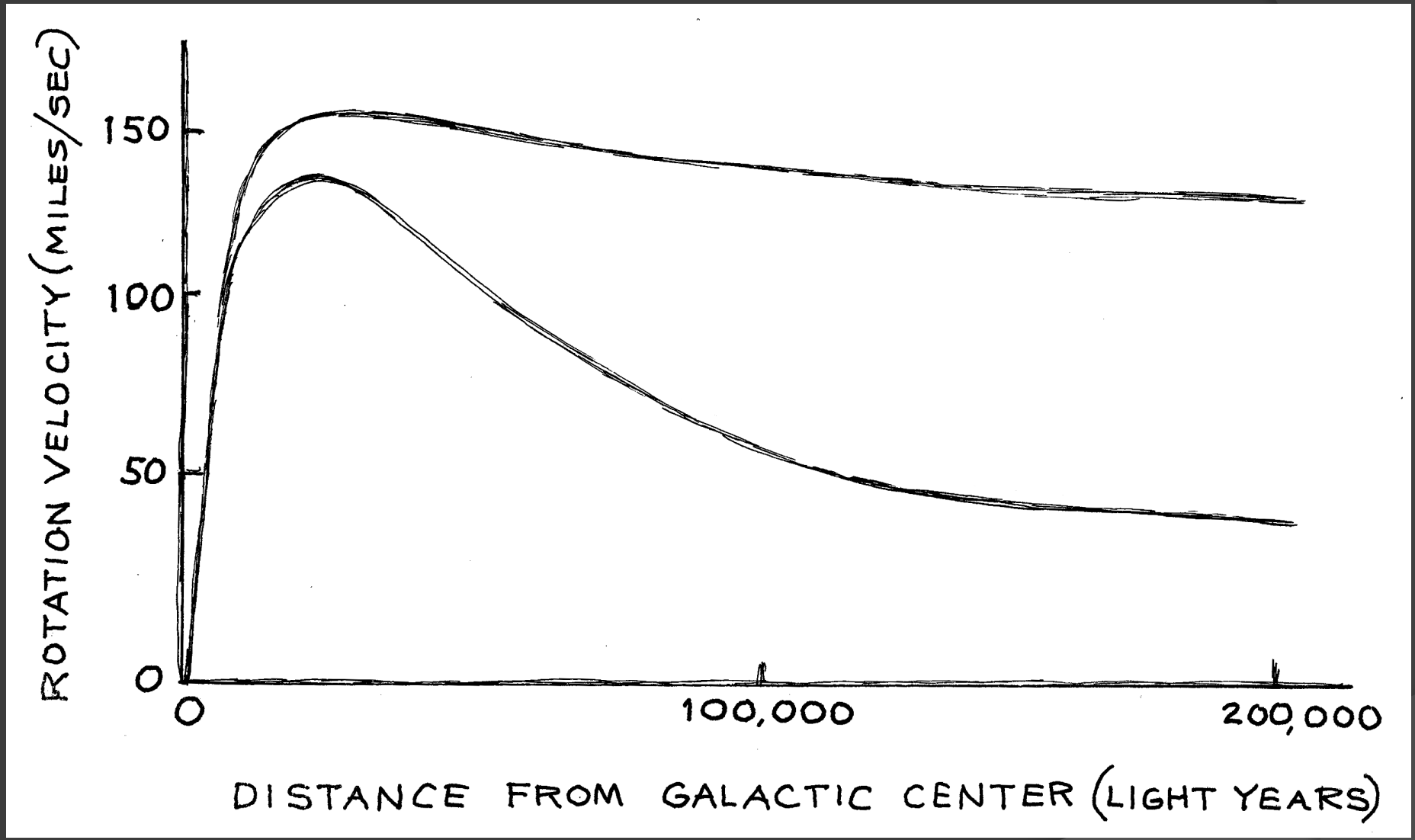


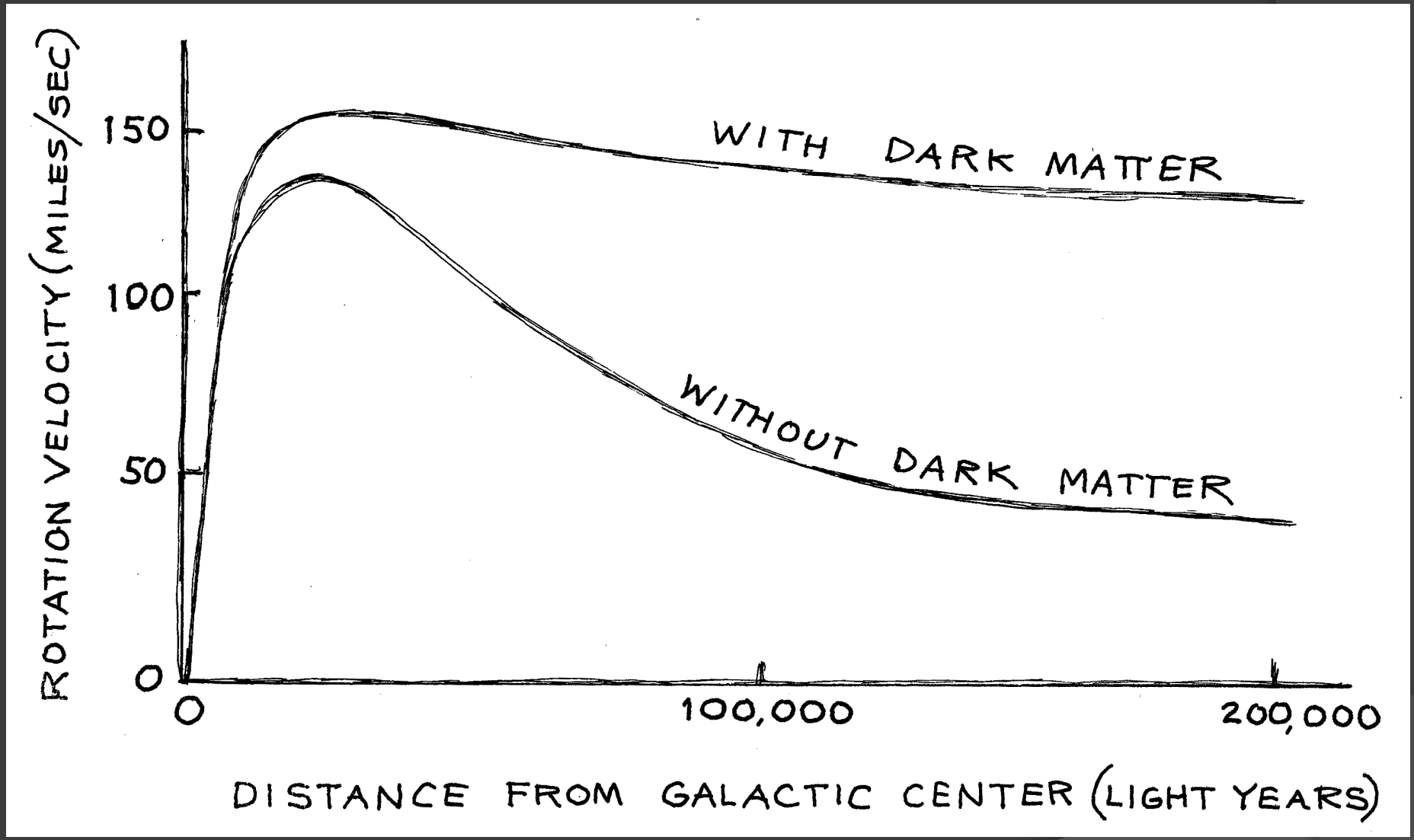
Early Pioneers

Fritz Zwicky

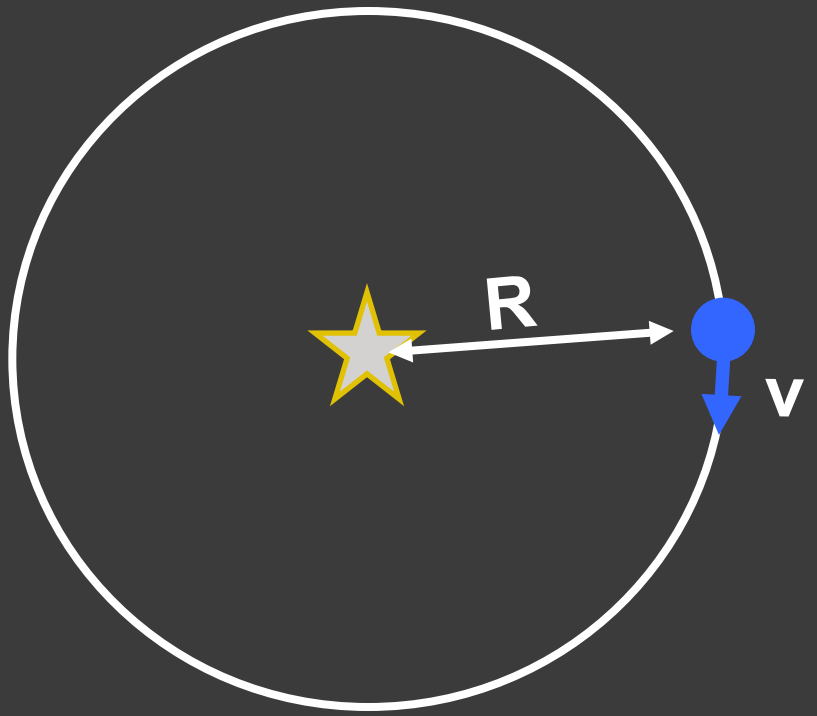
Vera Rubin


ROTATION VELOCITY (MILES/SEC)
150
100
50
0
0
100,000
200,000
DISTANCE FROM GALACTIC CENTER (LIGHT YEARS)

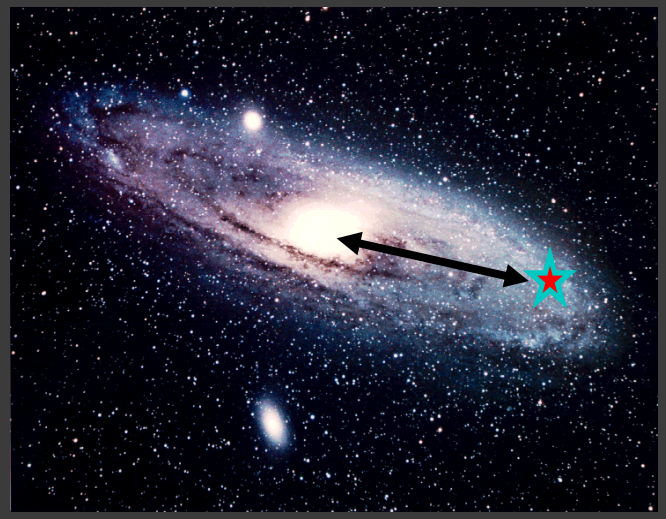

ROTATION VELOCITY (MILES/SEC)
150
100
50
0
0
100,000
200,000
DISTANCE FROM GALACTIC CENTER (LIGHT YEARS)



Most of the mass is invisible



$$\frac{v^2}{R} = \frac{G\,M_{Sun}}{R^2}$$



$$\frac{v^2}{R} = \frac{G\,M_{GALAXY}\,(<R)}{R^2}$$

Measure the speeds of stars $\Rightarrow$ Determine the mass of their galaxy

Three Possibilities

1) Galaxies are mostly made up of very non-luminous objects (black holes, neutron stars, white dwarf stars, large planets, etc.)

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1) Galaxies are mostly made up of very non-luminous objects (black holes, neutron stars, white dwarf stars, large planets, etc.)

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3) The missing mass consists of some other form of matter

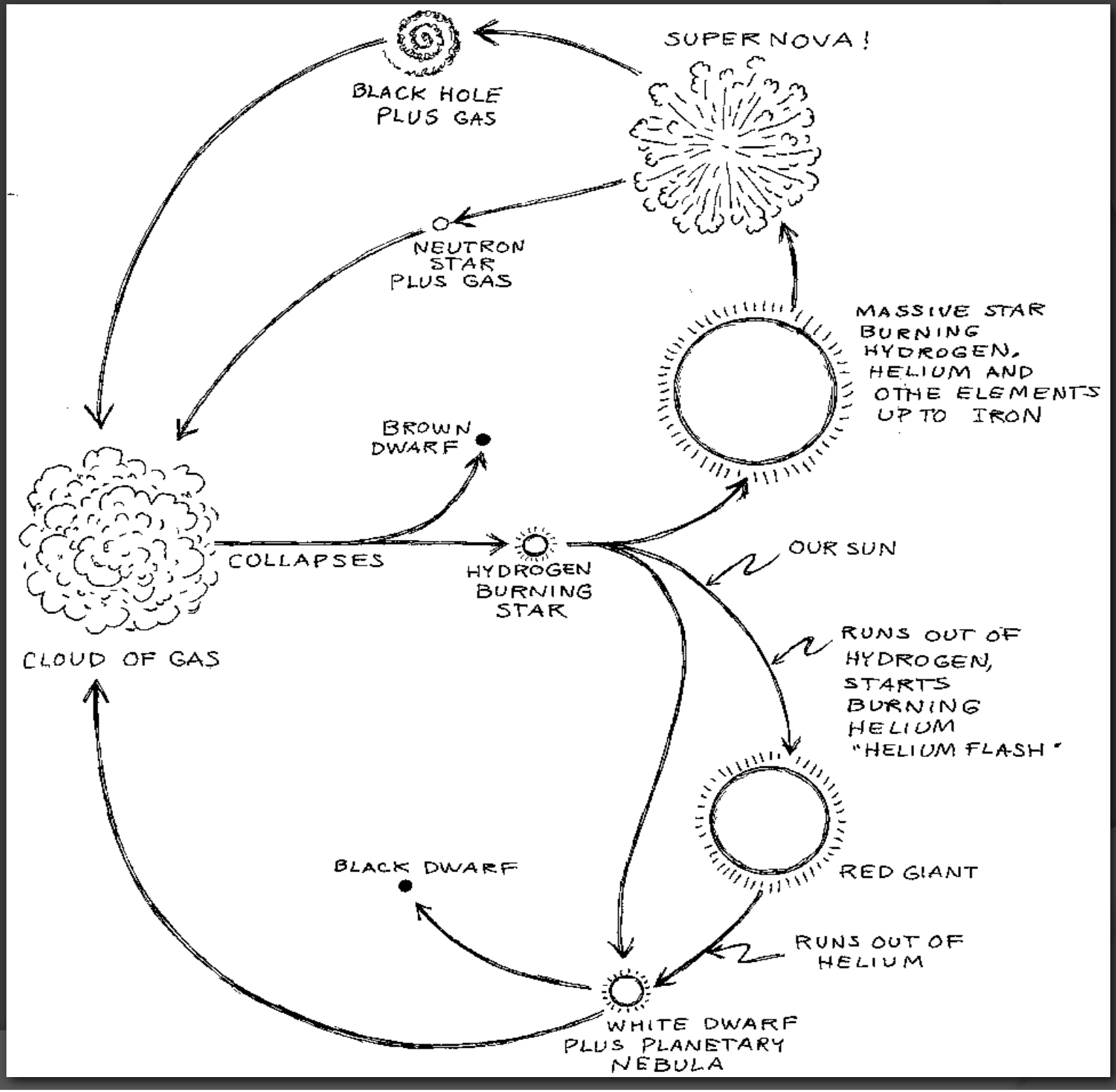
What Is Dark Matter?

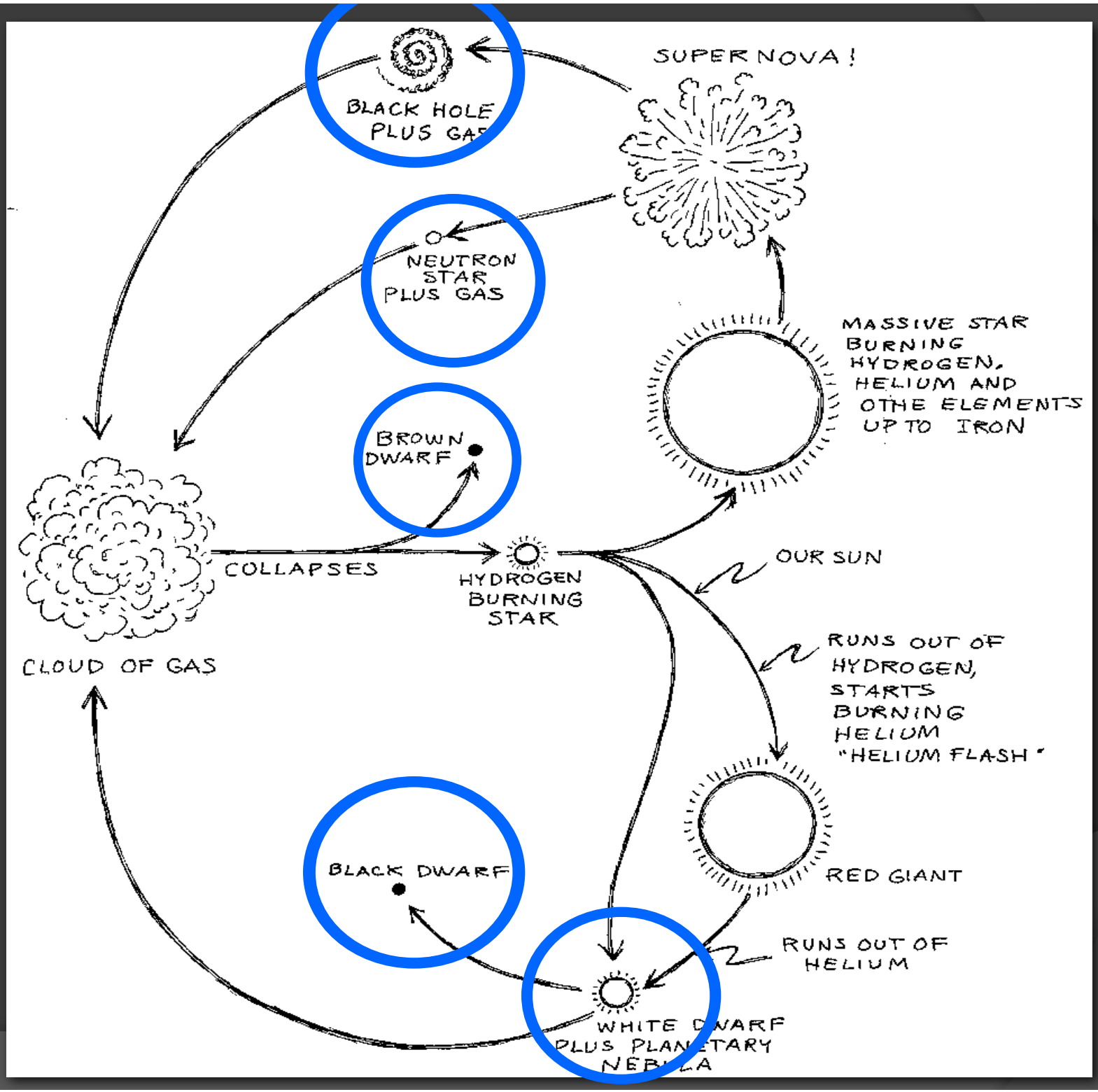
MACHOs

- Massive Compact Halo Objects
- Faint stars or planets (or rocks, baseballs, automobiles…)
- As most of the visible matter in our universe is made up of baryons (protons and neutrons), a first good guess is non-luminous or faint baryonic objects

WIMPs

- Weakly Interacting Massive Particles
- A gas of subatomic particles, interacting only through the weak force and gravity


SUPER NOVA !
BLACK HOLE PLUS GAS
NEUTRON STAR PLUS GAS
MASSIVE STAR BURNING HYDROGEN, HELIUM AND OTHE ELEMENTS UP TO IRON
BROWN DWARF
COLLAPSES
HYDROGEN BURNING STAR
OUR SUN
CLOUD OF GAS
RUNS OUT OF HYDROGEN, STARTS BURNING HELIUM "HELIUM FLASH"
RED GIANT
BLACK DWARF
RUNS OUT OF HELIUM
WHITE DWARF PLUS PLANETARY NEBULA


BLACK HOLE PLUS GAS
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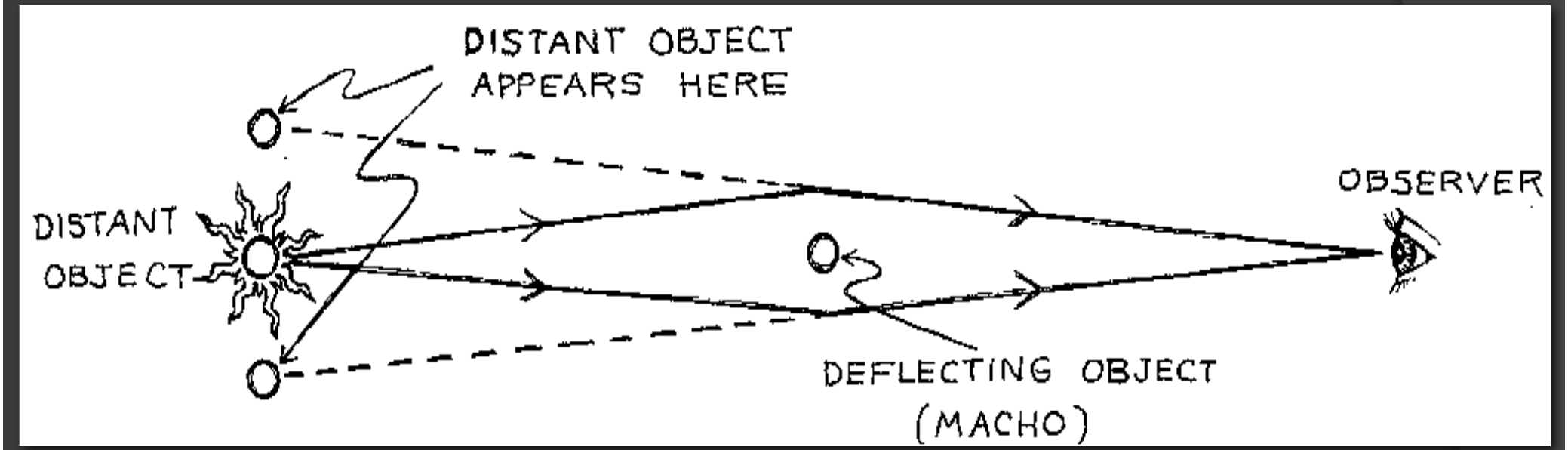
BLACK DWARF
RED GIANT
RUNS OUT OF HELIUM
WHITE DWARF PLUS PLANETARY NEBULA

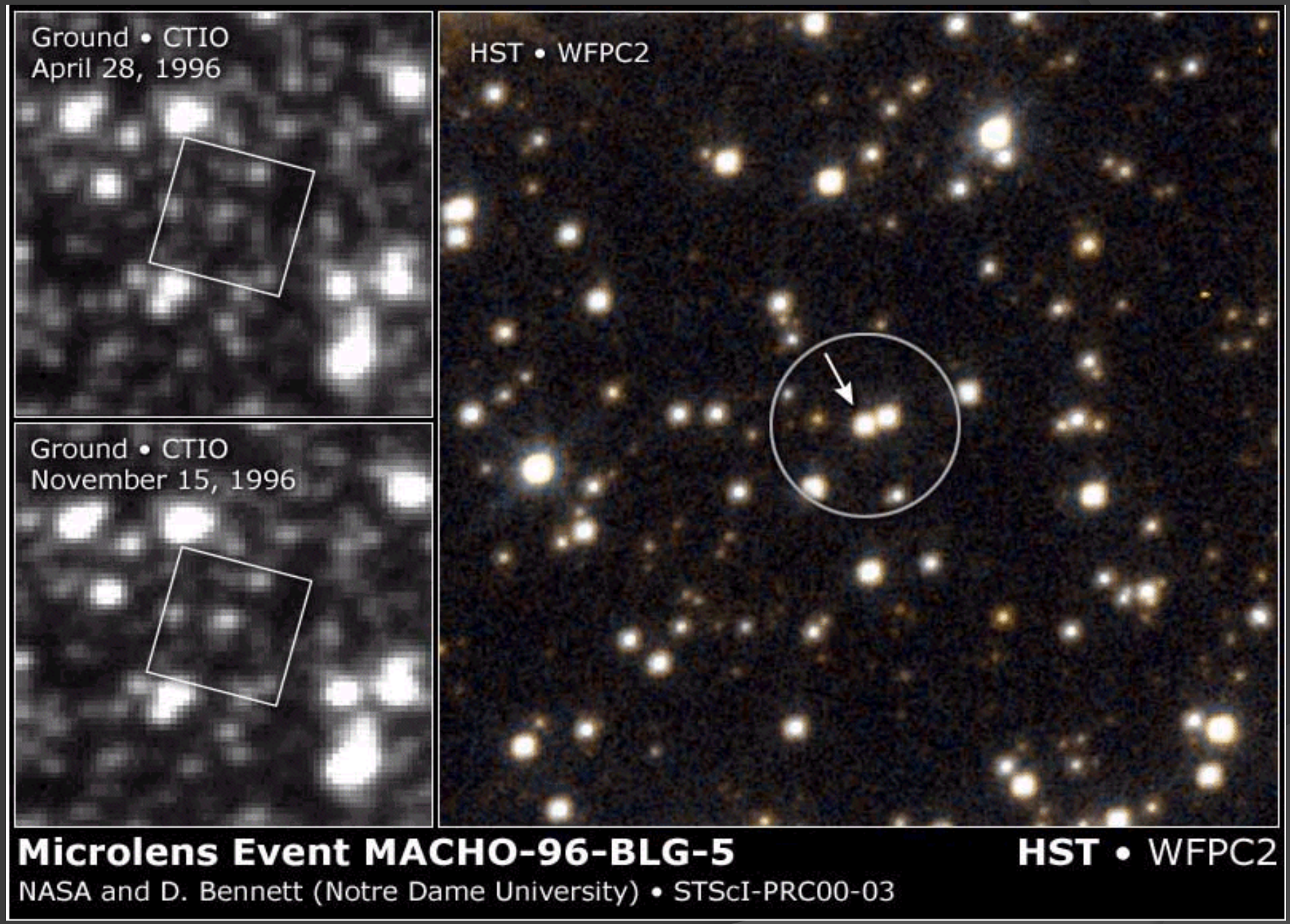
Faint Stars

- When most (~97%, including the Sun) stars run out of nuclear fuel, they compress into objects about the size of the Earth - white dwarfs
- White dwarfs become fainter as they age; most are 20% to 0.03% as luminous as the Sun
- When massive stars (~3%) run out of fuel, they explode as supernovae, leaving behind either a neutron star or a black hole
- Neutron stars are objects made up almost entirely of neutrons (few protons, electrons); they consist of a solar mass worth of material within a radius of ~10 kilometers
- Black holes are remnants of stars so massive that they collapse to a single point of space, with infinite density; nothing (including light) can escape their gravitational pull

"Seeing" Black Holes



Massive objects can be detected as gravitational lenses, even if they are themselves non-luminous



Caused by a black hole six times more massive than the Sun

Searches For Dark Matter With Gravitational Microlensing

- Although microlensing searches have found some faint and compact objects, they seem to be far too rare to make up much of the missing matter
- Whatever the dark matter is, it does not consist of compact objects with masses between about 3×10^{-8} and 50 solar masses
- MACHO dark matter also conflicts with the successful predictions of Big Bang nucleosynthesis, the cosmic microwave background, and large scale structure (primordial black holes being an exception)

Three Possibilities

1) Galaxies are mostly made up of very non-luminous objects (black holes, neutron stars, white dwarf stars, large planets, etc.)

2) Gravity does not work in galaxies or clusters the way it does on Earth or in our Solar System

3) The missing mass consists of some other form of matter

Modified Newtonian Dynamics (MOND)

- Begin by modifying Newtonian dynamics as follows:

$$F = ma \longrightarrow F = ma \times \mu(a)$$

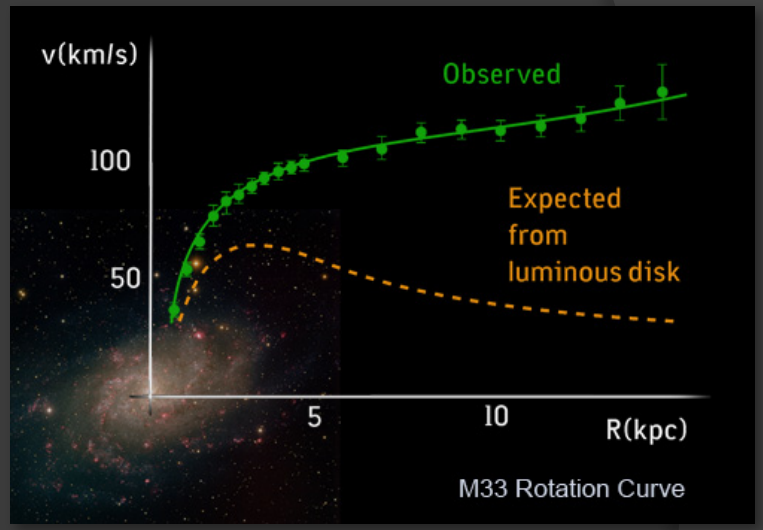
where μ≈1, except for small at acceleration, at which $\mu = a/a_0$

- For a circular orbit,

$$F = \frac{GMm}{r^2} = ma\mu$$

which in the low-acceleration limit yields

$$a = \frac{\sqrt{GMa_0}}{r} = \frac{v^2}{r} \quad \Longrightarrow \quad v = (GMa_0)^{1/4}$$



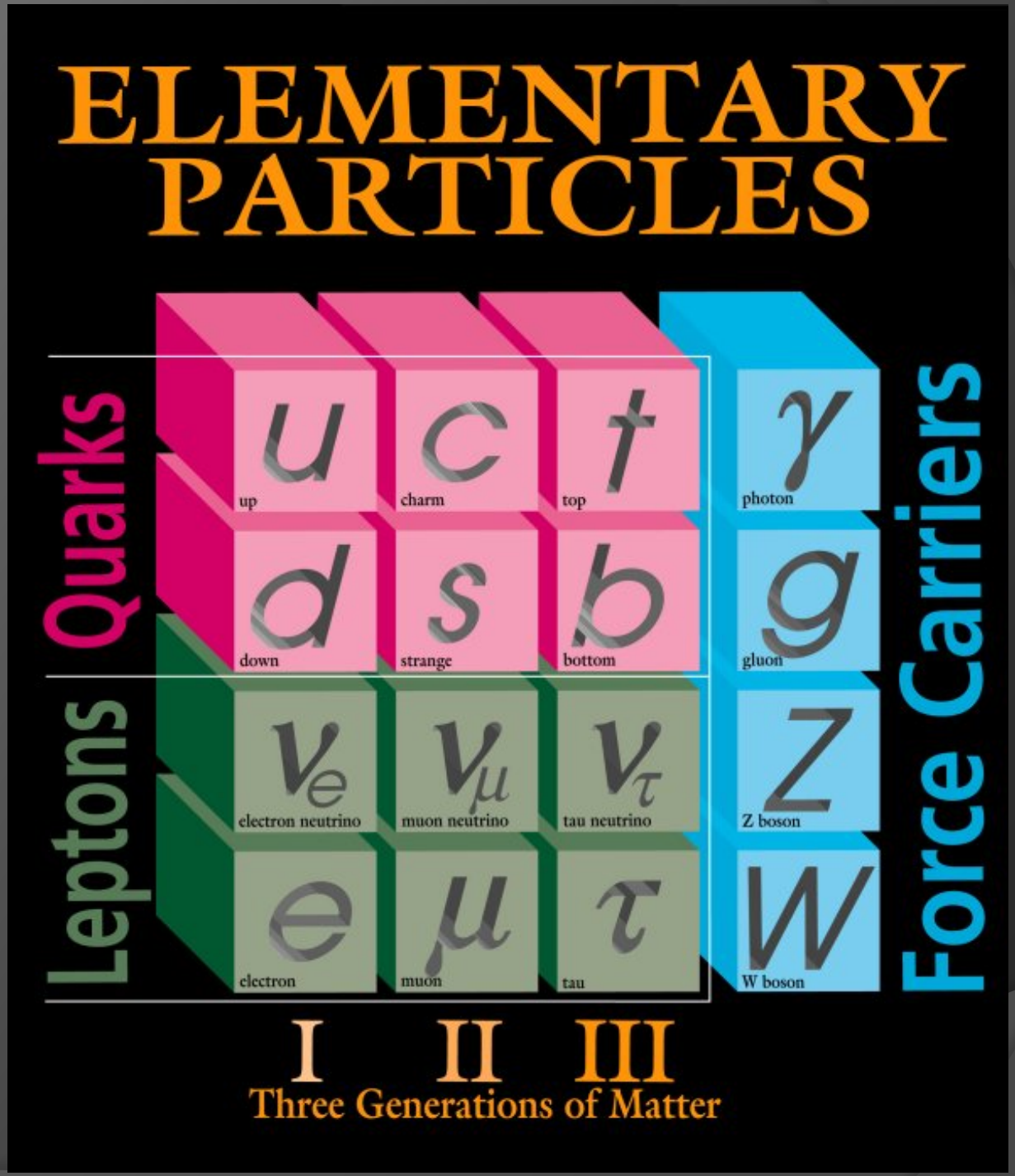
Rotational velocity independent of galactic radius (flat rotation curve)

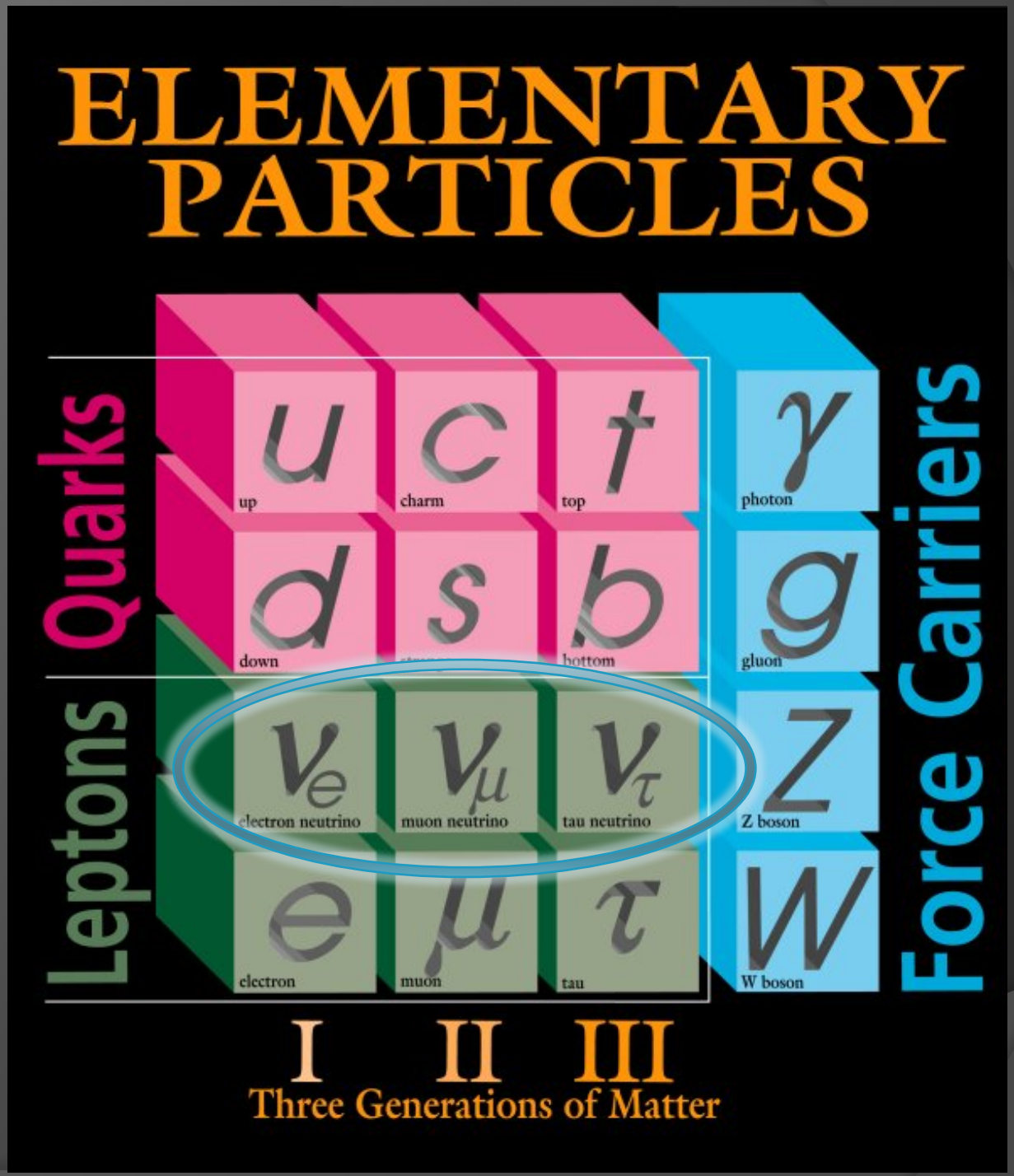
Modified Newtonian Dynamics (MOND)

- MOND has been quite successful in explaining the dynamics of galaxies, and provides an explanation for the Tully-Fisher relationship
- Galaxy clusters are not well described by MOND
- MOND is not consistent with cosmological measurements
- Dark matter suppresses the impact of baryon acoustic oscillations, explaining the observed matter power spectrum – MOND does not

Properties of Dark Matter

1) Not made of baryons (protons, neutrons)
2) Comes in “small” pieces (relative to stars, planets)
3) Does not significantly emit, reflect, or absorb light (electrically neutral)
4) Stable (or at least cosmologically long-lived)
5) Massive


ELEMENTARY PARTICLES
Quarks
Leptons
Force Carriers
u
up
c
charm
t
top
γ
photon
d
down
s
strange
b
bottom
g
gluon
νe
electron neutrino
νμ
muon neutrino
ντ
tau neutrino
Z
Z boson
e
electron
μ
muon
τ
tau
W
W boson
I
II
III
Three Generations of Matter

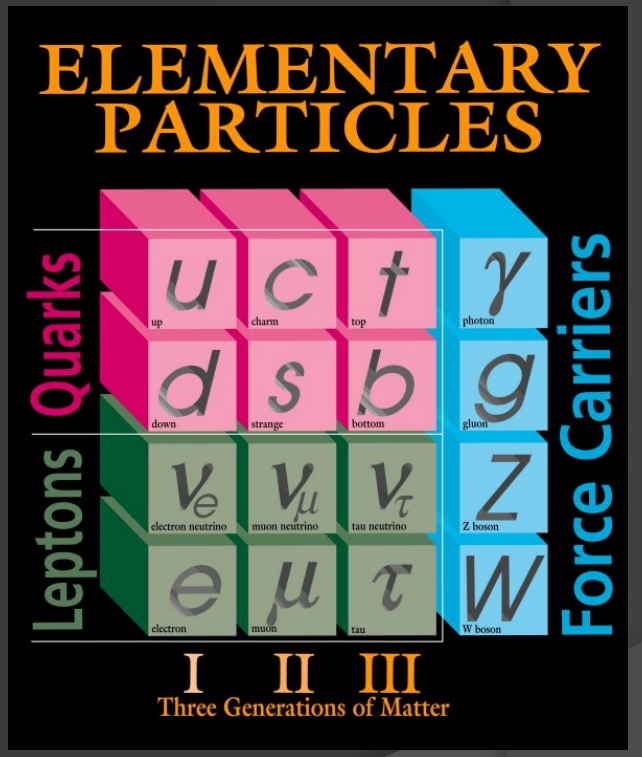

ELEMENTARY PARTICLES
Quarks
Leptons
Force Carriers
u
up
c
charm
t
top
γ
photon
d
down
s
b
bottom
g
gluon
νe
electron neutrino
νμ
muon neutrino
ντ
tau neutrino
Z
Z boson
e
electron
μ
muon
τ
tau
W
W boson
I
II
III
Three Generations of Matter

Cosmic Background Neutrinos

- Neutrinos were very abundant in the early universe, and some of them survived as a thermal relic
- Thermal decoupling and annihilation freeze-out happens at approximately the same time/temperature (t~2 sec, T~MeV)
- Assuming that neutrinos are stable, this should lead to a cosmic background of neutrinos with a similar density and temperature as the CMB (1.95 K vs 2.73 K)
- Indirect evidence of this background is observed in the anisotropies CMB

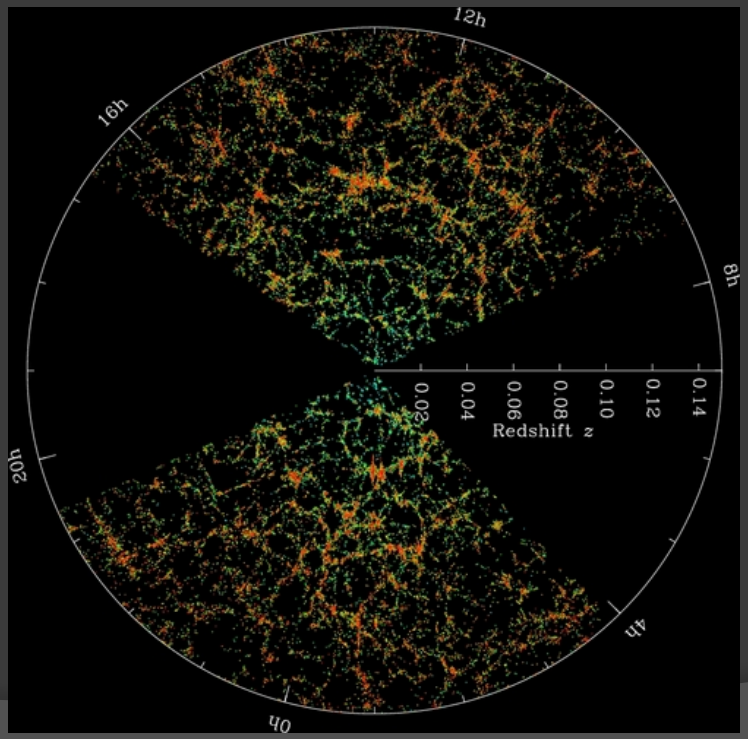
Could Neutrinos Be The Dark Matter?

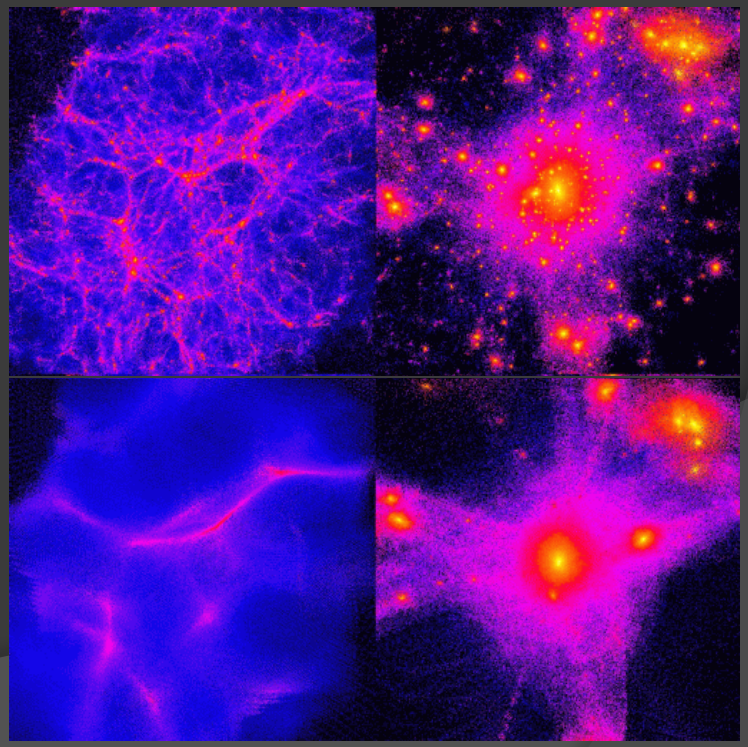
- The only weakly interacting massive particles in the Standard Model are neutrinos – might they make up the dark matter?
- Neutrinos decouple at ~MeV temperatures, and were thus relativistic when gravity began to bind dark matter into large scale structure
- Neutrinos therefore act as "hot" dark matter

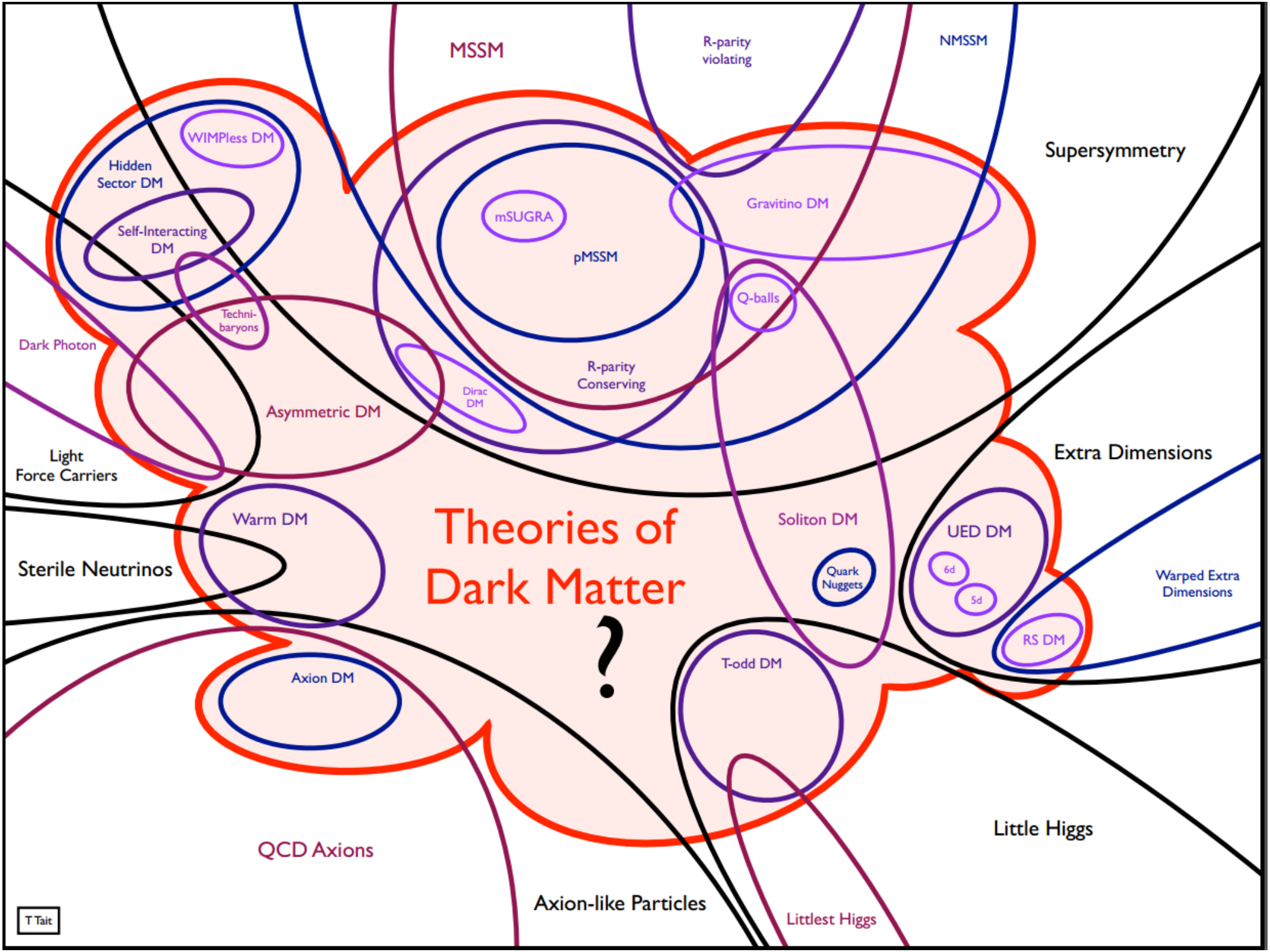


Hot and Cold Dark Matter

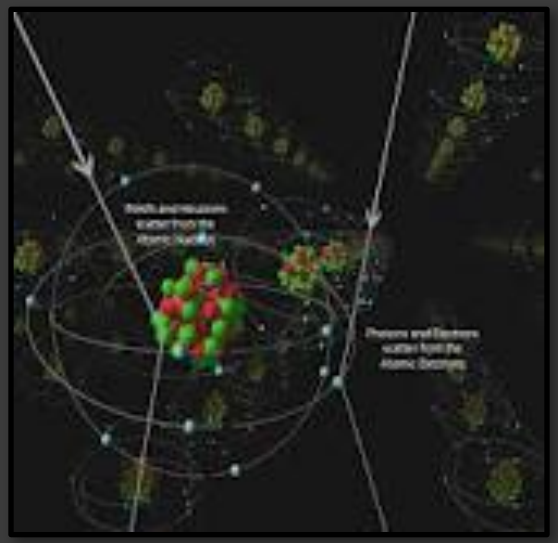
- Cosmologists use simulations to predict how dark matter will form structures under the influence of gravity
- The hotter the dark matter is, the less small scale structure forms, and the more "puffy" halos are
- When compared to observations, it is clear that dark matter is not hot – dark matter is not neutrinos

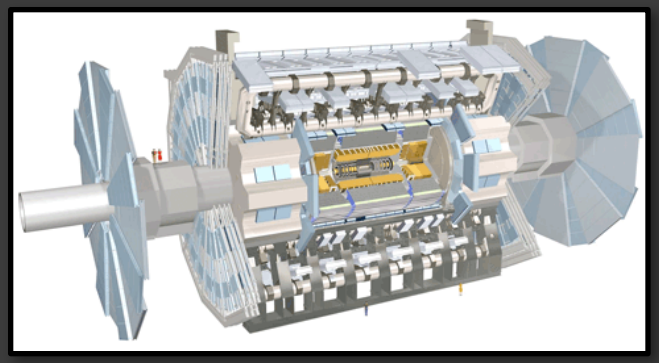


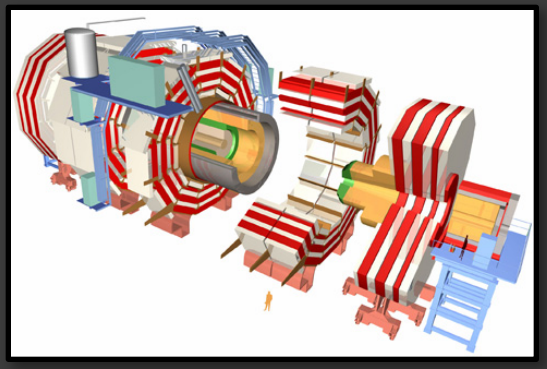


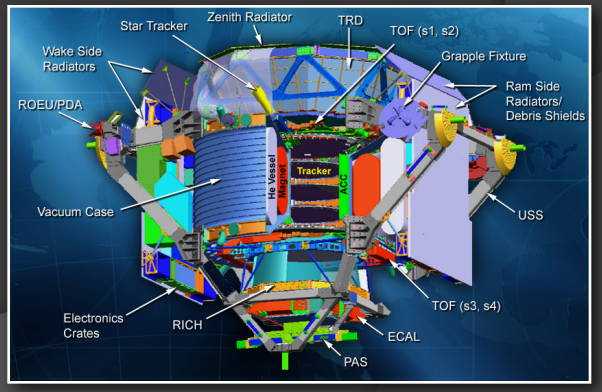

Theories of Dark Matter
?
MSSM
R-parity violating
NMSSM
Supersymmetry
WIMPless DM
Hidden Sector DM
Self-Interacting DM
mSUGRA
pMSSM
Gravitino DM
Q-balls
Techni-baryons
Dark Photon
R-parity Conserving
Dirac DM
Asymmetric DM
Light Force Carriers
Extra Dimensions
Warm DM
Sterile Neutrinos
Soliton DM
Quark Nuggets
UED DM
6d
5d
Warped Extra Dimensions
RS DM
T-odd DM
Axion DM
Little Higgs
QCD Axions
Axion-like Particles
Littlest Higgs
T Tait

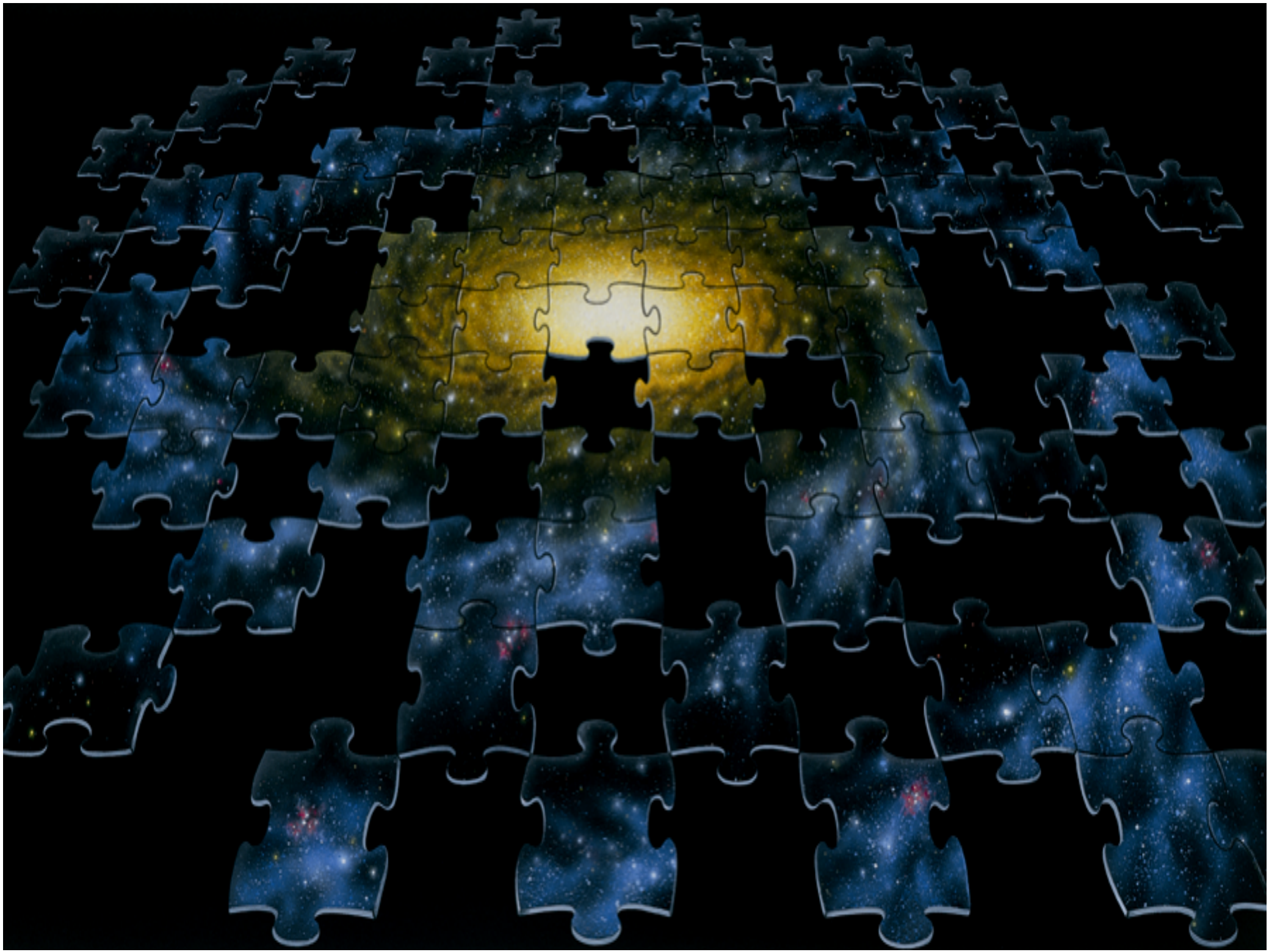
Putting the WIMP Hypothesis to the Test

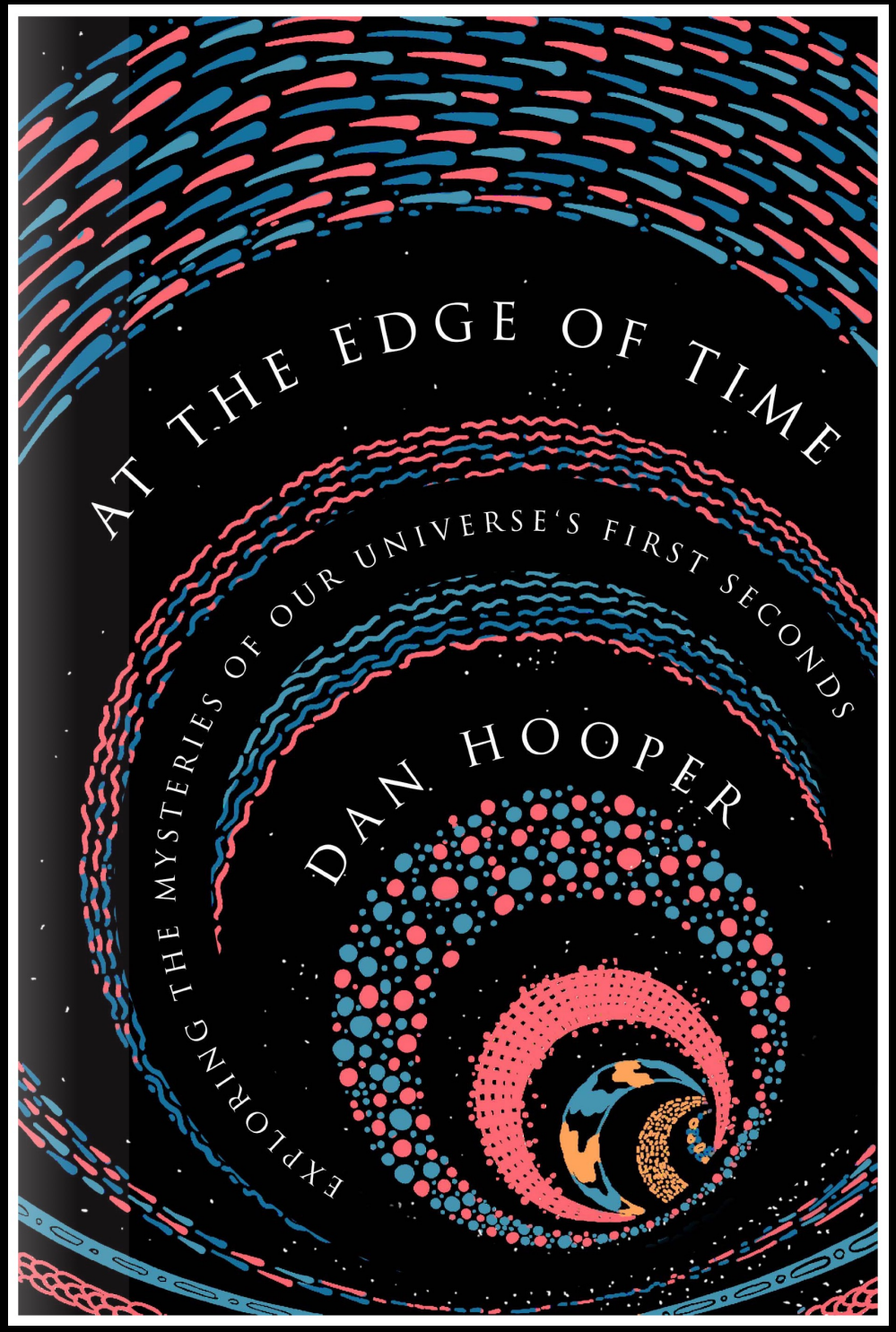











AT THE EDGE OF TIME
EXPLORING THE MYSTERIES OF OUR UNIVERSE'S FIRST SECONDS
DAN HOOPER